

2.5 17. Simplify. Express the answer in a + bi form.

$$\frac{(2+3i)(1+2i)}{(1-2i)(1+2i)} = \frac{2+4i+3i+6i^2}{1+2i-i-2i^2} = \frac{2+7i-6}{1+3} = \frac{-4+7i}{5} = \boxed{\frac{-4}{5} + \frac{7}{5}i}$$

PreCalculus B
Exam Review
ANSWERS

2.7 Find (if it exists) the a) as

18. $g(x) = \frac{x^2 - 9}{2x^2 - x - 15} =$

a) HA: $\frac{x^2}{2x^2} = \frac{1}{2}$ [

b) VA: $2x+5=0$

$$x = \frac{-5}{2}$$

Highlighted questions
are on your
review.

3.3 19. Simplify each expression.

(a) $\log_5 1 = \boxed{0}$

b/c $5^0 = 1$

(b) $\log \sqrt[4]{10} = x$

$$10^x = \sqrt[4]{10}$$

$$10^x = 10^{1/4}$$

$$x = \boxed{\frac{1}{4}}$$

(c) $3^{\log_3 7} = \boxed{7}$

3.4 20. Expand each logarithm:

(a) $\log_2 \left(\frac{8\sqrt[3]{x}}{y} \right)$

(b) $\log \left(\frac{\sqrt{x^5}}{10} \right)$

(c) $\ln(6x^4e^3)$

$\log_2 8 + \log_2 x^{\frac{1}{5}} - \log_2 y$

$$\boxed{3 + \frac{1}{5} \log_2 x - \log_2 y}$$

$\log x^{\frac{5}{2}} - \log 10$

$$\boxed{\frac{5}{2} \log x - 1}$$

$\ln b + \ln x^4 + \ln e^3$

$$\boxed{\ln b + 4 \ln x + 3}$$

Calculator Allowed.

→ loose 3% keep 97%

3.2 21. A radioactive isotope decays at a rate of 3% per day. A scientist has an initial amount of 50 g. Write a model for this situation. Determine approximately how many days it will take for half the isotope to decay.

$$y = 50(0.97)^x$$

where $x = \# \text{ of days}$
 $y = \text{amount of isotope}$

$$25 = 50(0.97)^x$$

$$0.5 = (0.97)^x$$

$$\log 0.5 = \log(0.97)^x$$

$$\log 0.5 = x \cdot \log 0.97$$

$$\frac{\log 0.5}{\log 0.97} = x$$

$$\boxed{x = 22.757 \text{ days}}$$

3.5 22. Solve algebraically:

(a) $\log_3 x + \log_3(x+8) = 2$

$$\log_3 [x(x+8)] = 2$$

$$3^2 = x(x+8)$$

$$9 = x^2 + 8x$$

$$0 = x^2 + 8x - 9$$

$$(x+9)(x-1) = 0$$

$$x+9=0 \quad x-1=0$$

$$x=9 \quad x=1$$

Not in domain

(b) $\log_2(x+5) - \log_2 x = 7$

$$\log_2 \left(\frac{x+5}{x} \right) = 7$$

$$2^7 = \frac{x+5}{x}$$

$$\frac{128}{1} = \frac{x+5}{x}$$

$$128x = x+5$$

$$127x = 5$$

$$x = \frac{5}{127}$$

(c) $3^{\frac{x}{2}} - 6 = 42$

$$3^{\frac{x}{2}} = 48$$

$$\log(3)^{\frac{x}{2}} = \log 48$$

$$\frac{1}{2} \cdot \log 3 = \log 48$$

$$x \cdot \log 3 = 2 \log 48$$

$$x = \frac{2 \log 48}{\log 3}$$

$$x = 7.047$$

(d) $-27 = -3 \cdot \left(\frac{1}{4}\right)^{6x}$

$$9 = \left(\frac{1}{4}\right)^{6x}$$

$$\log 9 = \log \left(\frac{1}{4}\right)^{6x}$$

$$\log 9 = 6x \cdot \log \left(\frac{1}{4}\right)$$

$$\frac{\log 9}{6 \log \left(\frac{1}{4}\right)} = x$$

$$-2.164 = x$$

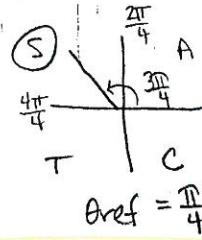
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
S	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
C	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
T	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Ch. 4, Ch. 5, 9.2, 9.4, 6.1 and 6.3 (2nd Semester) ~ No Calculator

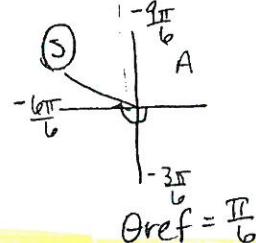
4.2 23. Find each exact value.

4.3

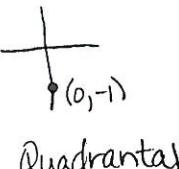
(a) $\cos\left(\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$



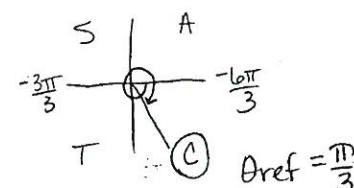
(b) $\sin\left(-\frac{7\pi}{6}\right) = \boxed{\frac{1}{2}}$



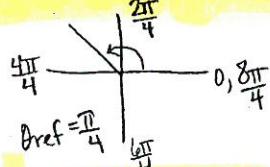
(c) $\tan\left(\frac{3\pi}{2}\right) = \boxed{\text{undefined}}$



(d) $\cos\left(\frac{-7\pi}{3}\right) = \boxed{\frac{1}{2}}$



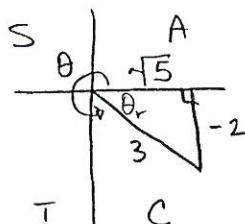
4.3 24. Find one positive angle and one negative angle that are coterminal with: $\frac{3\pi}{4}$.



$$\text{pos: } \frac{3\pi}{4} + \frac{8\pi}{4} = \frac{11\pi}{4}$$

$$\text{neg: } \frac{3\pi}{4} - \frac{8\pi}{4} = -\frac{5\pi}{4}$$

4.3 25. Given: $\sin \theta = -\frac{2}{3}$ and $\cos \theta > 0$. Find the values of the remaining five trigonometric functions of θ .



$$(-2)^2 + b^2 = 3^2$$

$$b^2 = 9 - 4$$

$$b = \sqrt{5}$$

$$\cos \theta = \frac{\sqrt{5}}{3}$$

$$\csc \theta = -\frac{3}{2}$$

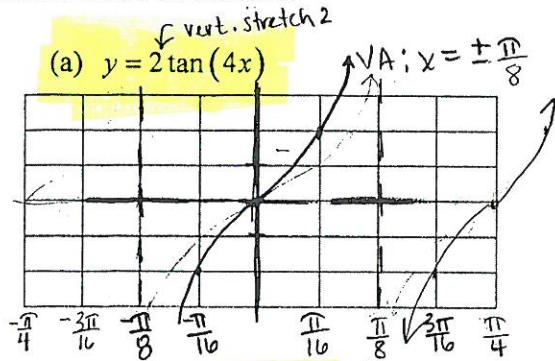
$$\sec \theta = \frac{3}{\sqrt{5}}$$

$$\tan \theta = -\frac{2}{\sqrt{5}}$$

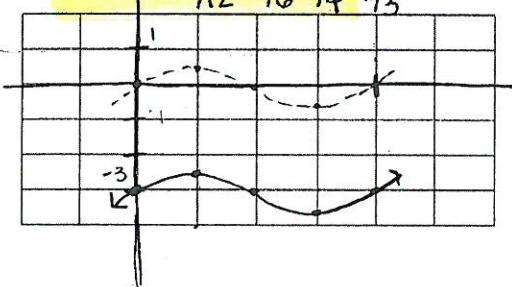
$$\cot \theta = -\frac{\sqrt{5}}{2}$$

- 4.4 26. State the amplitude, period, phase shift, and vertical shift, and then graph each function. Clearly label all tick marks on the axes.

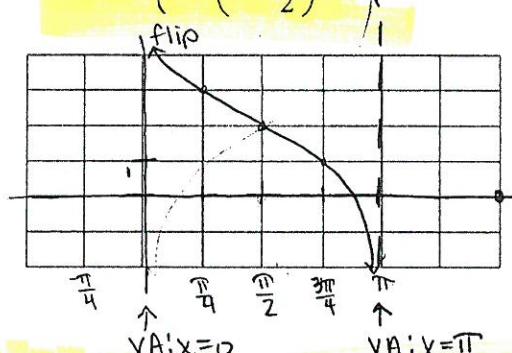
amp: n/a
per: $\frac{\pi}{4}$
ps: 0
vs: 0



amp: $\frac{1}{2}$
per: $\frac{\pi}{3}$
ps: 0
vs: -3



amp: n/a
per: pi
ps: $\frac{\pi}{2}$
vs: 2



- 4.4 27. Write an equation of the cosine function with amplitude = 2, period = $\frac{\pi}{2}$, phase shift = $-\frac{\pi}{8}$

and vertical shift = -3.

$$\text{per: } \frac{\pi}{2} \quad \frac{\pi}{2} = \frac{2\pi}{b}$$

$$\pi b = 4\pi$$

$$b = 4$$

- 4.4 28. Use the given graph...

- 5.3 (a) Write a sine function that fits the graph.

$$\text{ps: } -\frac{5\pi}{2} \quad y = 2 \sin\left(\frac{1}{5}(x + \frac{5\pi}{2})\right) + 4$$

- (b) Write a cosine function that fits the graph.

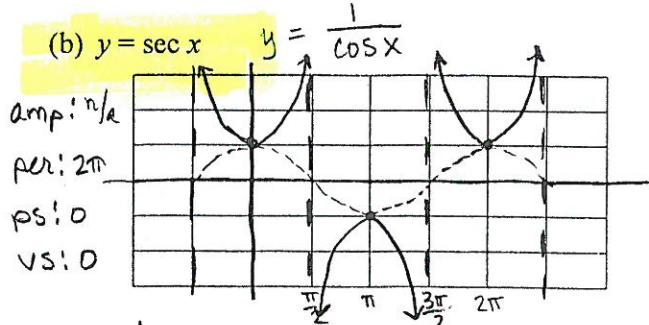
$$\text{ps: } 0 \quad y = 2 \cos\left(\frac{1}{5}x\right) + 4$$

- (c) Using identities, prove that the two equations you wrote are equal.

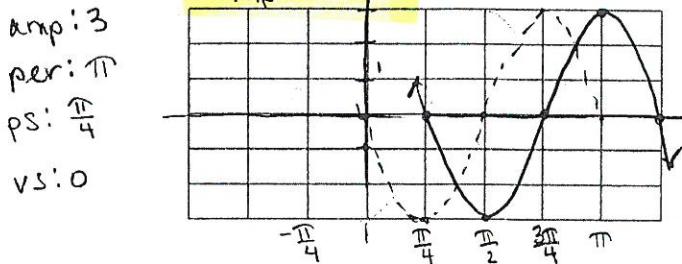
$$2 \sin\left(\frac{1}{5}(x + \frac{5\pi}{2})\right) + 4 = 2 \cos\left(\frac{1}{5}x\right) + 4$$

$$\text{LS} \rightarrow 2 \sin\left(\frac{1}{5}x + \frac{\pi}{2}\right) + 4$$

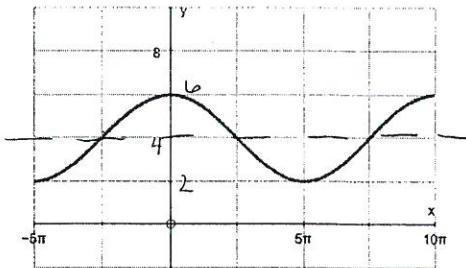
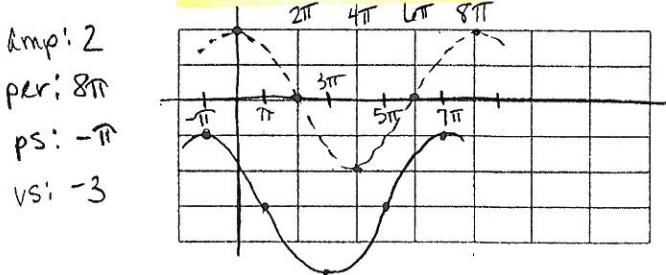
$$= 2 \left[\sin\left(\frac{1}{5}x\right) \cos\left(\frac{\pi}{2}\right) + \cos\left(\frac{1}{5}x\right) \sin\left(\frac{\pi}{2}\right) \right] + 4 = 2 \cos\left(\frac{1}{5}x\right) + 4 \rightarrow \text{RSV}$$



often missed
* (d) $y = -3 \sin\left(2x - \frac{\pi}{2}\right) = -3 \left[2(x - \frac{\pi}{4})\right]$



often missed
* (f) $y = 2 \cos\left(\frac{1}{4}x + \frac{\pi}{4}\right) - 3 = 2 \cos\left[\frac{1}{4}(x + \pi)\right] - 3$



} amp: 2 per: 10pi $\frac{2\pi}{b} = 10\pi$
vs: 4 $10\pi b = 2\pi$
 $b = \frac{1}{5}$

5.3 36. Find the exact value of $\cos 105^\circ$.

$$\begin{aligned}\cos 105^\circ &= \cos(45^\circ + 60^\circ) \\ &= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ \\ &= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}\end{aligned}$$

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
S	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
C	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
T	$\frac{1}{\sqrt{3}}$	1	$-\sqrt{3}$

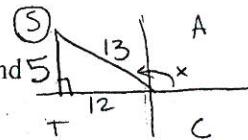
5.3- Rewrite using identities, then simplify if possible.

5.4

$$\begin{aligned}37. 1 - 2\sin^2 150^\circ &= \cos(2 \cdot 150^\circ) \\ &= \cos 300^\circ \\ &= \boxed{\frac{1}{2}}\end{aligned}$$

$\theta_{ref} = 60^\circ$

$$5.4 \quad 39. \text{ If } \cos x = -\frac{12}{13} \text{ and } x \text{ is in the second quadrant, find } \tan x$$



$$\begin{aligned}13^2 &= b^2 + 12^2 \\ 169 - 144 &= b^2 \\ 25 &= b^2\end{aligned}$$

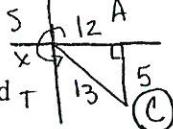
$$\begin{aligned}(a) \sin(2x) &= 2 \sin x \cos x \\ &= 2 \cdot \left(\frac{5}{13}\right) \cdot \left(-\frac{12}{13}\right) \\ &= \boxed{-\frac{120}{169}}\end{aligned}$$

$$(b) \tan(2x) = \frac{2 \tan x}{1 - \tan^2 x}$$

$$= \frac{2 \left(-\frac{5}{12}\right)}{1 - \left(-\frac{5}{12}\right)^2} = \frac{-\frac{10}{12}}{1 - \frac{25}{144}} = \frac{-\frac{10}{12}}{\frac{119}{144}} = \frac{120}{119}$$

$$5.4 \quad 40. \text{ If } \cot x = -\frac{12}{5} \text{ and } x \text{ is in the fourth quadrant, find } \cos(2x)$$

$$\begin{aligned}(a) \sin(2x) &= 2 \sin x \cos x \\ &= 2 \left(-\frac{5}{12}\right) \left(\frac{12}{5}\right) \\ &= \boxed{-\frac{120}{169}}\end{aligned}$$



$$\begin{aligned}(b) \cos(2x) &= \cos^2 x - \sin^2 x \\ &= \left(\frac{12}{13}\right)^2 - \left(-\frac{5}{12}\right)^2 \\ &= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}\end{aligned}$$

Ch5 41. Solve each equation for $[0, 2\pi]$.

$$(a) 2\sin^2 x = \sqrt{3} \sin x$$

$$2\sin^2 x - \sqrt{3} \sin x = 0$$

$$x = 0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$$

$$(b) 8\cos^2 x = 4$$

$$\begin{aligned}\cos^2 x &= \frac{1}{2} \\ \sqrt{\cos^2 x} &= \sqrt{\frac{1}{2}} \\ \cos x &= \pm \frac{1}{\sqrt{2}}\end{aligned}$$

$$\sin x (\sin x - \sqrt{3}) = 0$$

$$\begin{aligned}\sin x &= 0 \\ x &= 0, \pi\end{aligned}$$

$$\begin{aligned}\sin x - \sqrt{3} &= 0 \\ \sin x &= \frac{\sqrt{3}}{2}\end{aligned}$$

$$\begin{aligned}\theta_{ref} &= 60^\circ \\ x &= \frac{\pi}{3}, \frac{2\pi}{3}\end{aligned}$$

$$(c) \cos(2x) + \sin(x) = 0$$

use identity involving only sines!

$$1 - 2\sin^2 x + \sin x = 0$$

$$0 = 2\sin^2 x - \sin x - 1$$

$$0 = (2\sin x + 1)(\sin x - 1)$$

$$2\sin x + 1 = 0$$

$$\sin x = -\frac{1}{2}$$

$$x = \pi, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$(d) \sin(2x) - 2\cos(x) = 0$$

$$2\sin x \cos x - 2\cos x = 0$$

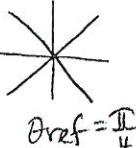
$$2\cos x (\sin x - 1) = 0$$

$$2\cos x = 0$$

$$\cos x = 0$$

$$\begin{aligned}\sin x - 1 &= 0 \\ \sin x &= 1 \\ x &= \frac{\pi}{2}, \frac{3\pi}{2}\end{aligned}$$

$$x = \frac{\pi}{2}$$



$$\theta_{ref} = \frac{\pi}{4}$$

Ch5 42. Solve for all values of x .

$$(a) \cos^2 x - 2\sin^2 x + 2 = 0$$

$$\cos^2 x - 2(1 - \cos^2 x) + 2 = 0$$

$$\cos^2 x - 2 + 2\cos^2 x + 2 = 0$$

$$3\cos^2 x = 0$$

$$\cos^2 x = 0$$

$$\sqrt{\cos^2 x} = \sqrt{0}$$

* Can also subst
for $\cos^2 x$ &
solve.

$$(b) \sin\left(\frac{3\pi}{2} - x\right) = 1$$

$$\sin\left(\frac{3\pi}{2}\right)\cos(x) - \cos\left(\frac{3\pi}{2}\right)\sin(x) = 1$$

$$-1 \cdot \cos(x) - 0 \cdot \sin(x) = 1$$

$$-\cos x = 1$$

$$\cos x = -1$$

$$x = \pi + 2\pi n$$



$$\cos x = 0 \leftarrow \text{No need for } \pm$$

$$x = \frac{\pi}{2} + 2\pi n$$

$$x = \frac{3\pi}{2} + 2\pi n$$

because $\pm 0 = 0$

Calculator Allowed

$\frac{1}{2}(0, -1)$

$\frac{1}{2}$

- 4.1 43. The wheel (including the tire) of a sports car under development by an auto company has an eleven inch radius. How many rpm's does the wheel make at 55 mph?



$$\frac{55 \text{ miles}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mile}} \times \frac{12 \text{ in}}{1 \text{ ft}} \times \frac{1 \text{ rev}}{2\pi/11 \text{ in}} \times \frac{1 \text{ hr}}{60 \text{ min}} = \frac{3484800}{(1320\pi)} = 840.338 \text{ rpm}$$

- 4.1 44. Find the measure of the intercepted arc in terms of π in a circle with diameter 60 inches and central angle of 72° .



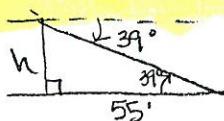
$$\frac{72^\circ}{180^\circ} \times \frac{\pi}{1} = \frac{2\pi}{5}$$

$$S = \theta r$$

$$S = \frac{2\pi}{5} (30 \text{ in})$$

$$S = 12\pi \text{ in}$$

- 4.6 45. The angle of depression from the top of a building to a point 55 feet away from the building (on level ground) is 39° . Determine the height of the building.

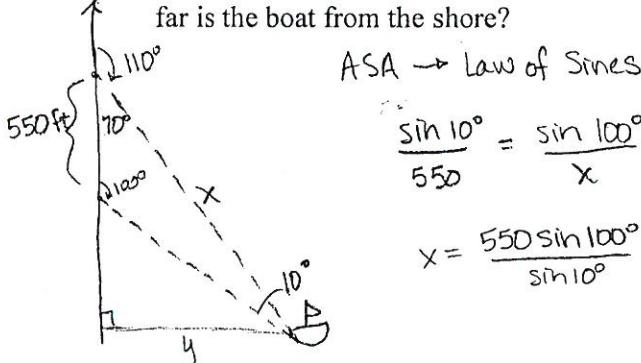


$$\tan 39^\circ = \frac{x}{55}$$

$$x = 55 \tan 39^\circ$$

$$x = 44.538 \text{ ft}$$

- 4.6 46. A shoreline runs north-south, and a boat is due east of the shoreline. The bearings of the boat from two points on the shore are 110° and 100° . Assume the two points are 550 feet apart. How far is the boat from the shore?



$$\sin 70^\circ = \frac{y}{x}$$

$$x \cdot \sin 70^\circ = y$$

$$\frac{550 \sin 100^\circ}{\sin 10^\circ} \cdot \sin 70^\circ = y$$

$$2931.094 = y$$

- 5.6 47. Find the area of each triangle.

$$(a) a = 7, b = 12, c = 13$$

$$\text{Semiperimeter: } \frac{1}{2}(7+12+13) = 16$$

$$A = \sqrt{16(16-7)(16-12)(16-13)}$$

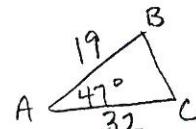
$$A = \sqrt{1728}$$

$$A = 41.569 \text{ units}^2$$

$$(b) A = 47^\circ, b = 32, c = 19$$

$$A = \frac{1}{2}(19, 32) \sin 47^\circ$$

$$A = 222.332 \text{ units}^2$$



5.5 48. Solve each triangle. If there are two Δ 's, solve both!

5.6

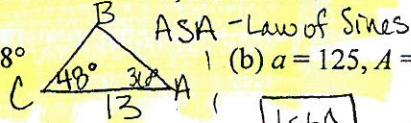
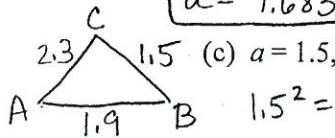
$$(a) A = 36^\circ, b = 13, C = 48^\circ$$

$$B = 96^\circ$$

$$\frac{\sin 96^\circ}{13} = \frac{\sin 36^\circ}{a}$$

$$a = \frac{13 \sin 36^\circ}{\sin 96^\circ}$$

$$a = 7.683$$



$$\frac{\sin 96^\circ}{13} = \frac{\sin 48^\circ}{c}$$

$$c = \frac{13 \sin 48^\circ}{\sin 96^\circ}$$

$$c = 9.714$$

1st Δ

$$\frac{\sin 25^\circ}{125} = \frac{\sin B}{150}$$

$$\sin B = \frac{150 \sin 25^\circ}{125}$$

$$B = \sin^{-1}(\downarrow)$$

$$B_1 = 30.474^\circ$$

$$C_1 = 124.526^\circ$$

$$\frac{\sin 25^\circ}{125} = \frac{\sin C}{c}$$

$$c = \frac{125 \sin C}{\sin 25^\circ}$$

$$C_2 = 243.679$$

2nd Δ * Find suppl.

of \angle we 1st found in 1st Δ ...

$$B_2 = 180 - B_1$$

$$B_2 = 149.526^\circ$$

$$C_2 = 5.474^\circ$$

$$\frac{\sin 25^\circ}{125} = \frac{\sin C_2}{c}$$

$$c = \frac{125 \sin C_2}{\sin 25^\circ}$$

$$C = 28.213$$

- 9.4 49. The sequence $\{2, 6, 18, 54, \dots\}$ is geometric. Find
 $r = 3$

(a) a recursive rule for the nth term. $a_1 = 2$

$$a_n = a_{n-1} \cdot 3 \text{ for } n \geq 2$$

(b) an explicit formula for the nth term.

$$a_n = 2 \cdot 3^{n-1}$$

- 9.4 50. Suppose an arithmetic sequence contains $a_{18} = 49$ and $a_{52} = 174.8$. Find ...

(a) the common difference

$$a_{18} + 34d = a_{52}$$

$$49 + 34d = 174.8$$

$$a_n = a_1 + 3.7(n-1)$$

$$49 = a_1 + 3.7(18-1)$$

(b) a_1

$$34d = 125.8$$

$$d = 3.7$$

$$49 = a_1 + 62.9$$

$$-13.9 = a_1$$

(c) a recursive formula

$$a_1 = -13.9$$

$$a_n = a_{n-1} + 3.7 \text{ for } n \geq 2$$

- 9.4 51. The fourth and ninth terms of a geometric sequence are 128 and 131072 respectively. Find

a) the common ratio

$$a_4 \cdot r^5 = a_9$$

$$128r^5 = 131072$$

$$a_n = a_1 \cdot 4^{n-1}$$

$$128 = a_1 \cdot 4^{n-1}$$

b) a_1

$$r^5 = 1024$$

$$128 = a_1 \cdot (64)$$

c) an explicit formula

$$r = 4$$

$$2 = a_1$$

$$a_n = 2(4)^{n-1}$$