…culus
…mester #2 Review

Name________________________________

Work must be shown in order to receive full (extra) credit for problems. (Section in which this was learned learned are listed in ***bold italics*** before each problem)

NO CALCULATOR

3.3 1. Simplify each expression.

a) $\log_5 1 = x = \boxed{0}$

$5^x = 1$

b) $\log \sqrt[4]{10} = x = \boxed{\frac{1}{4}}$

$10^x = 10^{1/4}$

c) $3^{\log_3 7} = \boxed{7}$

3.4 2. Expand each logarithm.

a) $\log_2\left(\frac{8\sqrt[5]{x}}{y}\right)$

$\log_2 8 + \log_2 x^{1/5} - \log_2 y$

$\boxed{3 + \frac{1}{5}\log_2 x - \log_2 y}$

b) $\log\left(\frac{\sqrt{x^5}}{10}\right)$

$\log x^{5/2} - \log 10$

$\boxed{\frac{5}{2}\log x - 1}$

c) $\ln(6x^4e^3)$

$\ln 6 + \ln x^4 + \ln e^3$

$\boxed{\ln 6 + 4\ln x + 3}$

CALCULATOR ALLOWED

3.2 3. A radioactive isotope decays at a rate of 3% per day. A scientist has an initial amount of 50g. Write a model for this situation. Determine approximately how many days it will take for half the isotope to decay.

-lose 3% keep 97%

$y = 50(0.97)^x$

$25 = 50(0.97)^x$

$0.5 = (0.97)^x$

$\log 0.5 = \log (0.97)^x$

$\log 0.5 = x \cdot \log(0.97)$

$x = \log 0.5 / \log 0.97$

$\boxed{x = 22.757 \text{ days}}$

3.3 4. Solve algebraically.

a) $\log_3 x + \log_3(x+8) = 2$

$\log_3 x(x+8) = 2$

$3^2 = x(x+8)$

$9 = x^2 + 8x$

$0 = x^2 + 8x - 9$

$0 = (x+9)(x-1)$

$\cancel{x = -9}$ $\boxed{x = 1}$

not in domain (extraneous)

b) $\log_2(x+5) - \log_2 x = 7$

$\log_2 \frac{x+5}{x} = 7$

$2^7 = \frac{x+5}{x}$

$128 = \frac{x+5}{x}$

$128x = x + 5$

$127x = 5$

$\boxed{x = \frac{5}{127}}$

c) $3^{\frac{x}{2}} - 6 = 42$

$3^{\frac{x}{2}} = 48$ —or→

$\log_3 48 = \frac{x}{2}$

$2 \cdot \log_3 48 = x$

$\boxed{7.047 = x}$

$3^{x/2} = 48$

$\log 3^{x/2} = \log 48$

$\frac{x}{2}\log 3 = \log 48$

$\frac{x}{2} = \frac{\log 48}{\log 3}$

$x = \frac{2\log 48}{\log 3} = \boxed{7.047}$

d) $-27 = -3 \cdot \left(\frac{1}{4}\right)^{6x}$

$9 = \left(\frac{1}{4}\right)^{6x}$ —or→

$\log_{1/4} 9 = 6x$

$\frac{\log_{1/4}(9)}{6} = x = \boxed{-.264}$

$\log 9 = \log\left(\frac{1}{4}\right)^{6x}$

$\log 9 = 6x \log \frac{1}{4}$

$\frac{\log 9}{6\log \frac{1}{4}} = x =$

NO CALCULATOR unit circle as well.

4.2 5. Find each exact value (remember reference triangles will help you).

a) $\cos\left(\frac{3\pi}{4}\right)$ $= -\frac{\sqrt{2}}{2}$

b) $\sin\left(-\frac{7\pi}{6}\right)$ $= \frac{1}{2}$

c) $\tan\left(\frac{3\pi}{2}\right)$ $=$ undef.

d) $\cos\left(\frac{-7\pi}{3}\right)$ $= \frac{1}{2}$

4.3 6. Find one positive angle and one negative angle that are coterminal with $\frac{3\pi}{4}$.

pos: $\frac{11\pi}{4}$

neg: $-\frac{5\pi}{4}$

4.3 7. Given: $\sin\theta = -\frac{2}{3}$ and $\cos\theta > 0$, find the values of the remaining five trigonometric functions of θ.
positive

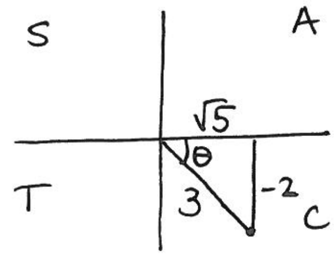
$\cos = \frac{\sqrt{5}}{3}$

$\tan = \frac{-2}{\sqrt{5}}$

$\csc\theta = \frac{-3}{2}$

$\sec\theta = \frac{3}{\sqrt{5}}$

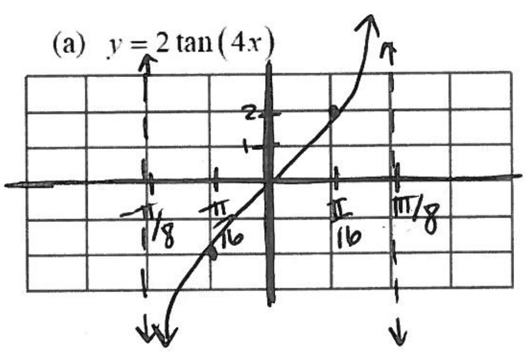
$\cot = \frac{\sqrt{5}}{-2}$



4.4 8. State the amplitude, period, phase shift, and vertical shift, and then graph each function. Clearly label all tick marks on the axes.

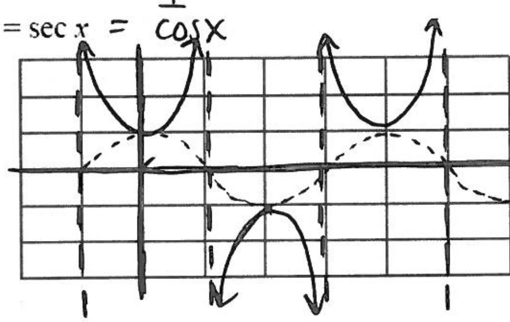
(a) $y = 2\tan(4x)$

amp: N/A
per: π/4
p.s. ∅
v.s. ∅
other: vert.str 2



(b) $y = \sec x = \frac{1}{\cos x}$

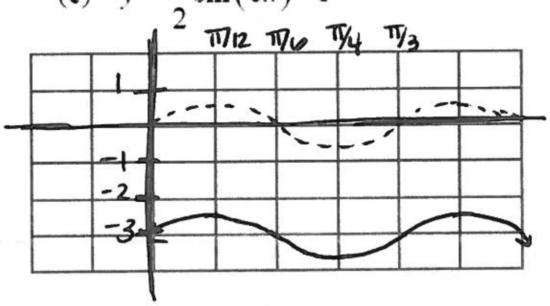
amp: NA
per: 2π
p.s. ∅
v.s. ∅



dashed graph is cosx

(c) $y = \frac{1}{2}\sin(6x) - 3$

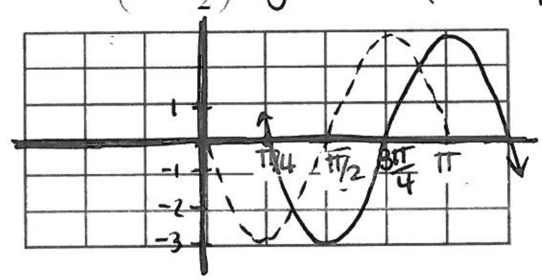
amp: 1/2
per: π/3
p.s. ∅
v.s. −3



★often missed★

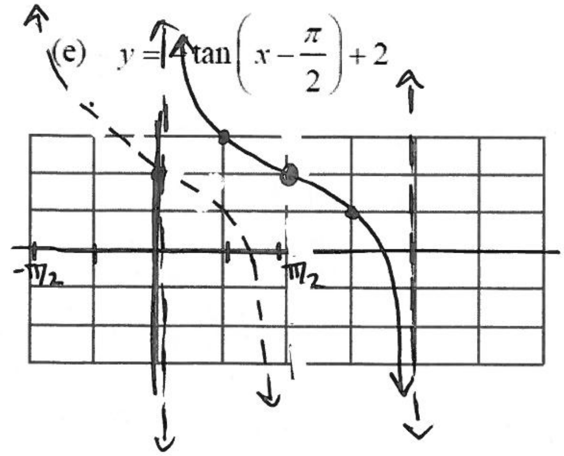
(d) $y = -3\sin\left(2x - \frac{\pi}{2}\right)$ $y = -3\sin(2(x - \frac{\pi}{4}))$

amp: 3
per: π
p.s. → $\frac{\pi}{4}$
v.s. ∅
flip



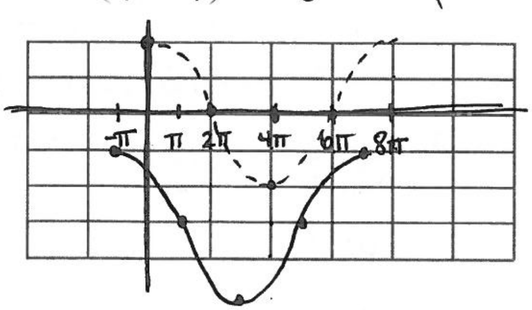
(e) $y = \boxed{4}\tan\left(x - \frac{\pi}{2}\right) + 2$

amp: N/A
per: π
p.s. $\rightarrow \frac{\pi}{2}$
v.s. $\uparrow 2$
other: flip



★often missed★
(f) $y = 2\cos\left(\frac{1}{4}x + \frac{\pi}{4}\right) - 3$ $y=2\cos\left(\frac{1}{4}(x+\pi)\right) - 3$

amp: 2
per: 8π
p.s. $\leftarrow \pi$
v.s. $\downarrow 3$
other: $\varnothing$

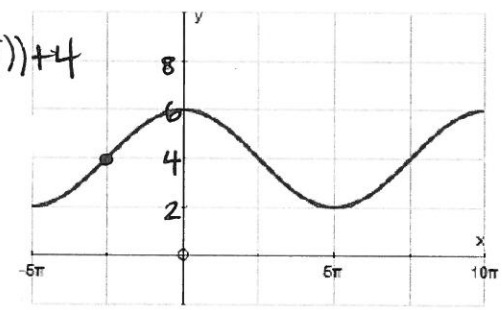


4.4 9. Write an equation of the cosine function with amplitude = 2, period = $\frac{\pi}{2}$, phase shift = $-\frac{\pi}{8}$ and vertical shift = -3.

$\frac{2\pi}{\text{per}} = \text{"b"}$ $\frac{2\pi}{\frac{\pi}{2}} = 2\pi \cdot \frac{2}{\pi} = 4$

$y = 2\cos\left(4\left(x + \frac{\pi}{8}\right)\right) - 3$

4.4 10. Use the given graph…



a) Write a sine function that fits the graph. $y = 2\sin\left(\frac{1}{5}\left(x - \frac{5\pi}{2}\right)\right) + 4$

amp: 2 p.s. $\leftarrow 2.5\pi$
per: 10π $\left(\frac{2\pi}{10\pi} = \frac{1}{5} = \text{"b"}\right)$ v.s. $\uparrow 4$

b) Write a cosine function that fits the graph.

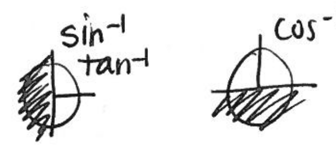
amp: 2 p.s. 0 $y = 2\cos\left(\frac{1}{5}x\right) + 4$
per: 10π $\left(\frac{1}{5} = \text{"b"}\right)$ v.s. $\uparrow 4$

c) Using identities, prove that the two equations you wrote are equal.

5.3 ★ $2\sin\left(\frac{1}{5}\left(x - \frac{5\pi}{2}\right)\right) + 4 = 2\cos\left(\frac{1}{5}x\right) + 4$ $\rightarrow 2\left(\cos\frac{1}{5}x\right) + 4$ ✓

$2\sin\left(\frac{1}{5}x - \frac{\pi}{2}\right) + 4$

$2\left[\sin\frac{1}{5}x \underbrace{\cancel{\cos\frac{\pi}{2}}}_{0} + \cos\frac{1}{5}x \cdot \underbrace{\cancel{\sin\frac{\pi}{2}}}_{1}\right] + 4$



4.7 11. Find each value. Remember domain restrictions!

a) $\arccos\left(\frac{1}{\sqrt{2}}\right) = \boxed{\frac{\pi}{4}}$

$\cos\theta = \frac{1}{\sqrt{2}}$

★remember $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$★

b) $\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$

$\cos\theta = -\frac{1}{2}$

c) $\sec\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)\right]$

$\sec\left(\frac{\pi}{3}\right) = \boxed{\frac{2}{1}}$

d) $\sin\left[\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

$\sin\left(\frac{-\pi}{6}\right) = \boxed{\frac{-1}{2}}$

e) $\cos\left[\arcsin(-1)\right]$

$\cos\left(\frac{-\pi}{2}\right) = \boxed{0}$

f) $\sin^{-1}\left[\sin\left(\frac{5\pi}{6}\right)\right]$

$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

★domain problem here!

5.1-5.4 Verify that each of the following is an identity. Be sure to show all steps.

*

12. $\sin^2\theta\left(\csc^2\theta-1\right)+\tan\left(-\theta\right)\cos\theta+\cos\left(\frac{\pi}{2}-\theta\right)=\cos^2\theta$

$\sin^2\theta\cdot\cot^2\theta-\tan\theta\cdot\cos\theta+\sin\theta=$

$\cancel{\sin^2\theta}\cdot\frac{\cos^2\theta}{\cancel{\sin^2\theta}}-\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}+\sin\theta=$

$\cos^2\theta-\cancel{\sin\theta}\cancel{+\sin\theta}=$

$\cos^2\theta=$ ✓

13. $\frac{\sin\beta}{\csc\beta}+\frac{\cos\beta}{\sec\beta}=1$

$\frac{\sin B}{\frac{1}{\sin B}}+\frac{\cos B}{\frac{1}{\cos B}}=$

$\sin B\cdot\frac{\sin B}{1}+\cos B\cdot\frac{\cos B}{1}=$

$\sin^2\beta+\cos^2\beta=$

$1=$ ✓

14. $\frac{2\tan x}{1+\tan^2 x}=\sin 2x$

$\frac{2\tan x}{\sec^2 x}=\frac{2\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}}=\frac{2\sin x}{\cancel{\cos x}}\cdot\frac{\cos^{\cancel{2}} x}{1}$

$=2\sin x\cdot\cos x$

$=\sin 2x$ ✓

15. $\frac{\cos x}{1+\sin x}+\frac{\cos x}{1-\sin x}=2\sec x$

16. $\sin\left(\pi-x\right)=\sin x$

5.3 17. Find the exact value of $\cos 105^\circ$.

$\cos(105)=\cos(135-30)$

$=\cos 135\cos 30+\sin 135\sin 30$

$=-\frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\ \frac{1}{2}$

$-\frac{\sqrt{6}}{4}+\frac{\sqrt{4}}{4}=\boxed{\frac{-\sqrt{6}+\sqrt{4}}{4}}$

OR $\cos(45+60)$

$\cos 45\cos 60-\sin 45\sin 60$

$\frac{\sqrt{2}}{2}\ \frac{1}{2}-\frac{\sqrt{2}}{2}\ \frac{\sqrt{3}}{2}$

$\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}=\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

5.3-5.4 Rewrite using identities, then simplify if possible.

18. $1-2\sin^2 150^\circ$

$(1-2\sin^2 A=\cos 2A)$

$\cos(2\cdot 150)$

$\cos(300)$

$\boxed{\frac{1}{2}}$

19. $\sin\left(\frac{\pi}{5}\right)\cos\left(\frac{\pi}{2}\right)+\cos\left(\frac{\pi}{5}\right)\sin\left(\frac{\pi}{2}\right)$

$\sin A\cos B+\cos A\sin B=\sin(A+B)$

$\sin\left(\frac{\pi}{5}+\frac{\pi}{2}\right)$ or $\sin(36+90)$

$\sin\left(\frac{2\pi}{10}+\frac{5\pi}{10}\right)$ $\boxed{\sin(126)}$

$\boxed{\sin\left(\frac{7\pi}{10}\right)}$ or

CALCULATOR ALLOWED

4.1 20. The wheel (including the tire) of a sports car under development by an auto company has an eleven inch radius. How many rpm's (revolutions per minute) does the wheel make at 55 mph?

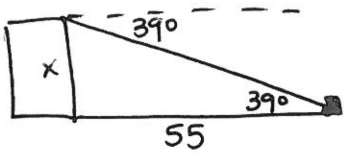
$r=30$

4.1 21. Find the measure of the intercepted arc in terms of π in a circle with diameter 60 inches and central angle of 72°.

$S=\theta\cdot r$

$72\cdot\frac{\pi}{180}=\frac{2\pi}{5}$ $\quad S=\frac{2\pi}{5}\cdot 30=\boxed{12\pi}$

5.6 22. The angle of depression from the top of a building to a point 55 feet away from the building (on level ground) is 39°. Determine the height of the building.



$\tan 39^\circ=\frac{x}{55}$ $\quad x=44.54$

5.6 23. Find the area of each triangle.

a) $a=7, b=12, c=13$ $\quad A=\sqrt{s(s-a)(s-b)(s-c)}$

$s=\frac{7+12+13}{2}=16$

$A=\sqrt{16(16-7)(16-12)(16-13)}$

$A=\sqrt{1728}=\boxed{41.569u^2}$

b) $A=47^\circ, b=32, c=19$ $\quad A=\frac{1}{2}ab\sin C$

$=\frac{1}{2}bc\sin A$

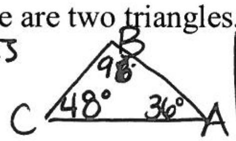
$\frac{1}{2}(32)(19)\sin 47$

$\boxed{=222.332\text{ units}^2}$

5.5 24. Solve each triangle. If there are two triangles, solve both! acute∡ across shorter $=2\Delta s$!

5.6 $2\measuredangle s\rightarrow$ LAW OF SINES

a) $A=36^\circ, b=13, C=48^\circ$

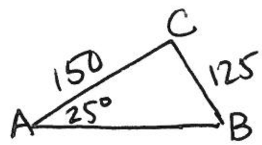


$\frac{\sin 96}{13}=\frac{\sin 36}{a}$ $\quad a=7.683$

$\frac{\sin 96}{13}=\frac{\sin 48}{c}$ $\quad c=9.714$

$a=7.683$ $A=36^\circ$
$b=13$ $B=96^\circ$
$c=9.714$ $C=48^\circ$

b) $a=125, A=25^\circ, b=150$ $\quad$ SSA



$\frac{\sin 25}{125}=\frac{\sin B}{150}$ $\quad \boxed{B=30.474}$

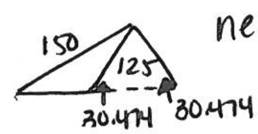
$180-30.474-25=\boxed{124.53=C}$

$\frac{\sin 25}{125}=\frac{\sin 124.53}{c}$

$\boxed{c=243.679}$

$a=125$ $A=25^\circ$
$b=150$ $B=30.47^\circ$
$c=243.68$ $C=124.53^\circ$

new $\angle B=180-30.474=$ $\boxed{B=149.526^\circ}$



$\frac{\sin 25}{125}=\frac{\sin 5.47}{c}$

$a=125$ $A=25^\circ$
$b=150$ $B=149.53^\circ$
$c=28.21$ $C=5.47^\circ$

c) $a=1.5, b=2.3, c=1.9$ $\quad$ LAW OF COSINES

$1.5^2=2.3^2+1.9^2-2(2.3)(1.9)\cos A$

$2.25=8.9-8.74\cdot\cos A$

$-8.9 \quad -8.9$

$-6.65=-8.74\cdot\cos A$

$.76087=\cos A$

$\cos^{-1}(.76087)=\boxed{40.459^\circ=A}$

$\frac{\sin 40.459}{1.5}=\frac{\sin C}{2.3}$ $\quad \boxed{B=84.261^\circ}$

$\boxed{C=55.280^\circ}$