

#1-9 PRECALC

1. What is $\sqrt{-1}$? i

Just because we refer to solutions as imaginary does not mean that the solutions are meaningless. Fields such as quantum mechanics and electromagnetism depend on the mathematics of imaginary numbers. When engineers design airplane wings or cell-phone towers, imaginary numbers are vital to their calculations. Applications abound in electrical engineering, vibration engineering, polymer science, navigation, and more. Applications abound in the real world, touching our lives via design of popular features such as the vibrating ringer in our cell phones or the bass boosters in our MP3 players. More heavy duty applications include the design of missile guidance systems. While we are not ready to dive into these topics yet, we are ready to learn to simplify the square root of negative numbers correctly.

2. Evaluate the following expressions: (no calculator)

a) $\sqrt{-25} = 5i$

b) $\sqrt{-60} = \sqrt{60}i = 2\sqrt{15}i = 2i\sqrt{15}$

$\sqrt{60}$
 $\begin{matrix} \wedge & \wedge \\ 6 & 10 \\ \textcircled{2}3 & \textcircled{2}5 \end{matrix}$

c) $\sqrt{(-6)^2 - 4(3)(-2)}$
 $\sqrt{36 + 24}$
 $\sqrt{60} = 2\sqrt{15}$
 $\begin{matrix} \wedge & \wedge \\ 30 & 2 \\ 15 & 2 \end{matrix}$

d) $\sqrt{5^2 - 4(-6)(-3)}$
 $\sqrt{25 - 72}$
 $\sqrt{-47} = i\sqrt{47}$

When adding, subtracting and multiplying complex numbers, pay close attention to whether you are adding, subtracting, or multiplying. MOST people that miss these questions do so because they decide to multiply everything.

4. What is i^2 ? $i^2 = \sqrt{-1} \cdot \sqrt{-1} = \sqrt{-1}^2 = -1$ multiplying by i^2 is like multiplying by -1 .

5. Perform the indicated operation and simplify each of the following expressions:

a) $(7-3i) + (6-i)$
 $13-4i$

b) $(7-3i) - (6-i) = 7-3i-6+i$
 $1-2i$

b) $(7-3i)(6-i)$
 $42-7i-18i+3i^2$
 $42-25i-3 = 39-25i$

d) $(5i-3)(2i+1)$
 $10i^2+5i-6i-3$
 $-10-i-3 = -13-i$

6. Find the conjugate of each complex number then multiply them together.

a) $(7+5i)(7-5i)$
 $49-35i+35i-25i^2$
 $49+25 = 74$

b) $(-5-3i)(-5+3i)$
 $25-15i+15i-9i^2$
 $25+9 = 34$

When you multiply a complex number by its conjugate the answer is a REAL number.

Handwritten notes and diagrams at the bottom of the page, including a box containing $\frac{a+bi}{c+di} = \frac{(a+bi)(c-di)}{(c+di)(c-di)}$ and other algebraic manipulations.

In order to divide complex numbers, you will multiply the numerator and denominator by the conjugate of the denominator so that the denominator will be a real number. ♪: If you came directly from regular Algebra 2, you may never have seen this ... please do not hesitate to ask for some help! ☺

Example: $\frac{5-2i}{7+5i} = \frac{(5-2i)(7-5i)}{(7+5i)(7-5i)}$ multiply top and bottom by conjugate of bottom

$$= \frac{35 - 25i - 14i + 10i^2}{49 - 35i + 35i - 25i^2}$$

$$= \frac{35 - 39i - 10}{49 + 25}$$

$$= \frac{25 - 39i}{74} \leftarrow \text{real \# (no } i)$$

$$= \frac{25}{74} - \frac{39}{74}i$$

$a + bi$

$(2+3i)(5+7i)$
 $(5-7i)(5+7i)$

8. Divide the following complex numbers. Write your answer in the form $a + bi$.

a) $\frac{2+8i}{-5-3i} \cdot \frac{(-5+3i)}{(-5+3i)}$

$$= \frac{-10 + 6i - 40i + 24i^2}{25 - 15i + 15i - 9i^2}$$

$$= \frac{-10 - 34i - 24}{34} = \frac{-34 - 34i}{34} = \frac{-34}{34} - \frac{34i}{34} = \boxed{-1 - i}$$

b) $\frac{2+3i}{9-4i} \cdot \frac{(9+4i)}{(9+4i)} = \frac{18 + 8i + 27i + 12i^2}{81 + 36i - 36i - 16i^2}$

$$= \frac{18 + 35i - 12}{81 + 16} = \frac{6 + 35i}{97}$$

$$= \boxed{\frac{6}{97} + \frac{35}{97}i}$$

9. Solve the following quadratic equations by using the quadratic formula.

a) $x^2 + x + 11 = 5x - 8$

$$x^2 - 4x + 19 = 0$$

$$x = \frac{+4 \pm \sqrt{16 - 4(1)(19)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 - 76}}{2}$$

$$= \frac{4 \pm \sqrt{-60}}{2} = \frac{4 \pm \sqrt{60}i}{2}$$

$$= \frac{4}{2} \pm \frac{2i\sqrt{15}}{2}$$

$$= \boxed{2 \pm i\sqrt{15}}$$

b) $2x^2 - 12x = -25$

$$2x^2 - 12x + 25 = 0$$

$$x = \frac{12 \pm \sqrt{144 - 4(2)(25)}}{2(2)}$$

$$= \frac{12 \pm \sqrt{144 - 200}}{4}$$

$$= \frac{12 \pm \sqrt{-56}}{4}$$

$$= \frac{12 \pm 2i\sqrt{14}}{4}$$

$$= \frac{12}{4} \pm \frac{2i\sqrt{14}}{4} = \boxed{3 \pm \frac{\sqrt{14}}{2}i}$$

Solving quadratic equations can also be done by using completing the square. Completing the square is most useful when the coefficient of x^2 is equal to 1 (and the coefficient of x is even).

... "Go back to Section P5 or 2.1 if you need a refresher on how to complete the square."

10. Solve the following equation by completing the square and compare it to question 9a above:

$$x^2 + x + 11 = 5x - 8$$

11. The impedance of an electrical current is a way of measuring how much the circuit impedes the flow of electricity. The impedance can be a complex number. A circuit is being designed that must have an impedance that satisfies the function $f(x) = 2x^2 - 12x + 40$, where x is a measure of the impedance. Find the zeros of the function. (Use any method you like).

$$2(x^2 - 6x + 20)$$

$$2(\quad)(\quad)$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{12 \pm \sqrt{-176}}{4}$$

$$\frac{12 \pm i\sqrt{176}}{4}$$

$$\frac{12 \pm 4i\sqrt{11}}{4}$$

$3 \pm i\sqrt{11}$

$\sqrt{176}$
 $\swarrow \quad \searrow$
 $4 \quad 44$
 $\swarrow \quad \searrow$
 $4 \quad 11$

12. Draw a picture of (or explain why you are not able to draw) each of the following:

a) a quadratic function having only one real number root.



b) a quadratic function having only one complex root.

Complex must occur in pairs

c) a quadratic function with two real roots.



d) a quadratic function with two complex roots. no real



13. Previously, we used synthetic division to show that a value of x was indeed a zero of a function. If you were told that one of the solutions to the equation $x^2 - 4x + 5 = 0$ is $x = 2 + i$, use synthetic division to show that it really is a zero. Using the result of your synthetic division, what is the other solution?

$2+i$

$$\begin{array}{r|rrr} & 1 & -4 & 5 \\ & \downarrow & 2+i & -5 \\ \hline & 1 & -2+i & 0 \end{array}$$

$(X - (2+i))$

$(2+i)(-2+i) = -4 + 2i - 2i + i^2 = -4 - 1 = -5$

$x = 2+i$
 $x = 2-i$

♫: You will use synthetic division with complex numbers in lesson 2.6

In the last example the conjugates $x = 2 + i$ and $x = 2 - i$ were both solutions. This is not a coincidence. In a polynomial if $x = a + bi$ is a zero, then the conjugate $x = a - bi$ is also a zero. Using the last example, this means that the expression $x^2 - 4x + 5$ could be factored to $[x - (2 + i)][x - (2 - i)]$.

14. Write the polynomial function of minimum degree in standard form whose given zeros and their multiplicities include those listed below.

a) $x = 4i$ $x = -4i$

$$(x - 4i)(x + 4i)$$

$$x^2 + 4ix - 4ix - 16i^2$$

$$x^2 - 16(-1)$$

$$x^2 + 16$$

b) $x = -5$ and $x = 4i$

$$(x + 5)(x - 4i)(x + 4i)$$

$$(x + 5)(x^2 + 16)$$

$$x^3 + 16x + 5x^2 + 80$$

$$x^3 + 5x^2 + 16x + 80$$

- c) $x = -2$ (multiplicity 2), and another zero of $x = 2 + 3i$

$$(x + 2)(x + 2)(x - 2 + 3i)(x - 2 - 3i)$$

$$x^2 + 2x + 2x + 4 \quad x^2 - 2x - 3ix - 2x + 4 + 6i + 3ix - 6i - 9i^2 + 9$$

$$(x^2 + 4x + 4) \cdot (x^2 - 4x + 13)$$

OPTIONAL EXTRA PRACTICE:

For more practice on each of the following topics...refer to pages 234-235 of your book.

Given zeros of a polynomial, find the equation of the polynomial... #5-15

Pre Calculus
Worksheet 2.6

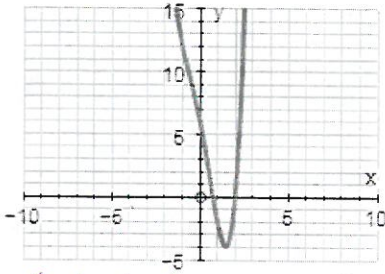
1. State the Fundamental Theorem of Algebra in your own words. Why do we need it?

An n^{th} degree polynomial has n zeros. Some are real, some can be complex.

2. Use the equations and the graphs below to do the following:

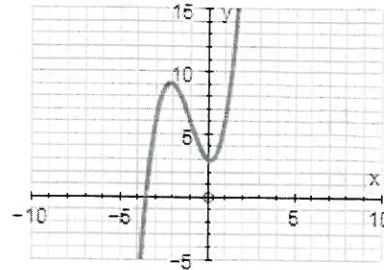
- Identify the number of complex zeros each equation has according to the Fundamental Theorem of Algebra.
- Determine how many real and non-real zeros each function has according to the graph.

a) $f(x) = x^4 - 2x^2 - 7x + 6$



4 total zeros
2 real zeros
pair \rightarrow 2 non-real zeros

b) $f(x) = x^3 + 3x^2 - x + 3$



3 total zeros
2 nonreal
1 real

3. How many real zeros are you guaranteed to have if you have an odd degree polynomial?

at least one real zero

4. Is it possible to find a polynomial $f(x)$ that has the degree 4 with the given zeros? Explain why or why not.

a) $-3, 1 + 2i$ and $1 - i$

$1 - 2i$ $1 + i$

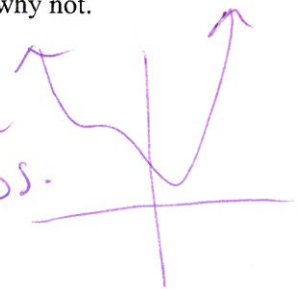
Five zeros

impossible to be degree 4.

b) $1 - 3i$ and $4 + i$

$1 + 3i$ $4 - i$

4 nonreal zeros.



5. Draw a picture of (or explain why you are not able to draw) each of the following:

a) a quartic function having only one complex root.

Complex (nonreal) roots must occur in pairs.

b) a quartic function with two real roots.



2 real
2 imaginary

$$(2+i)(-2+i) = -4 + 2i - 2i + i^2 = -4 - 1 = -5$$

$$(18-9i)(2+i) = 36 + 18i - 18i - 18i^2 = 36 + 18$$

[No Calculator] Using the given zero, find all complex zeros of the polynomial function.

6. $f(x) = 5x^4 + 3x^3 + 3x^2 + 3x - 2$; zeros: $x = -1$ and $\frac{2}{5}$

$$\begin{array}{r|rrrrr} -1 & 5 & 3 & 3 & 3 & -2 \\ & & -5 & 2 & -5 & 2 \\ \hline & 5 & -2 & 5 & -2 & \cancel{0} \\ \frac{2}{5} & & \downarrow & 2 & 0 & 2 \\ \hline & 5 & 0 & 5 & \cancel{0} & \\ \hline & & & & & (5x^2 + 5) \text{ is factor} \end{array}$$

$$5x^2 + 5 = 0$$

$$5x^2 = -5$$

$$x^2 = -1$$

$$x = \pm i, -1, \frac{2}{5}$$

7. $h(x) = x^4 - 4x^3 - 4x^2 + 36x - 45$; zero: $x = 2 + i$

$$\begin{array}{r|rrrrr} 2+i & 1 & -4 & -4 & 36 & -45 \\ & & 2+i & -5 & -18-9i & 45 \\ \hline & 1 & -2+i & -9 & 18-9i & \cancel{0} \\ & & 2-i & 0 & -18+9i & \\ \hline & 1 & 0 & -9 & \cancel{0} & \\ \hline & & & & & (x^2 - 9) = 0 \\ & & & & & (x+3)(x-3) = 0 \\ & & & & & x = 3 \quad x = -3 \\ \hline & & & & & x = 2 \pm i, \pm 3 \end{array}$$

8. Use factoring to find all the zeros of $f(x) = x^3 + 3x^2 + 4x + 12$

Start with factor by grouping

$$x^3 + 3x^2 + 4x + 12$$

$$x^2(x+3) + 4(x+3)$$

$$(x^2+4)(x+3)$$

$$x^2 + 4 = 0$$

$$x^2 = -4$$

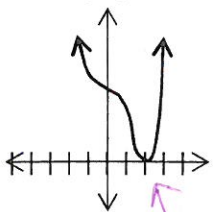
$$x = \pm 2i$$

$$x = \pm 2i, -3$$

$$x + 3 = 0$$

$$x = -3$$

9. Use the graph below to help factor $g(x) = 3x^4 - 11x^3 + 10x^2 - 4x + 8$. The scale on the x-axis is 1 unit / tick.



real zero at 2
bounce!
multiplicity 2

$$\begin{array}{r|rrrrr} 2 & 3 & -11 & 10 & -4 & 8 \\ & & 6 & -10 & 0 & -8 \\ \hline & 3 & -5 & 0 & -4 & \cancel{0} \\ 2 & & 6 & 2 & 4 & \\ \hline & 3 & 1 & 2 & \cancel{0} & \end{array}$$

$$(x-2)^2(3x^2 + x + 2)$$

irreducible quadratic factor

**Pre Calculus
Worksheet 2.7 Day 1**

#1-10

No Calculator should be used on this worksheet.

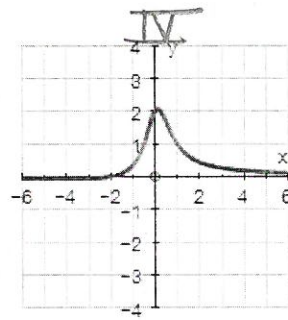
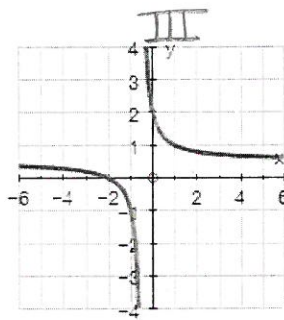
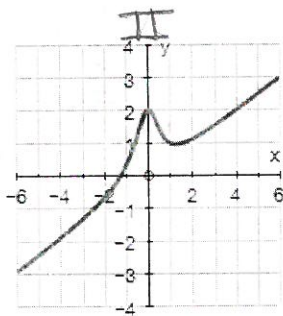
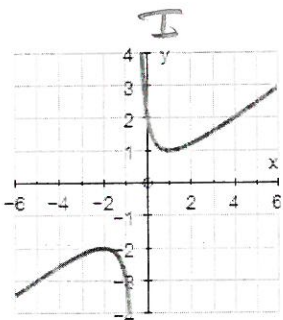
Match the function with the corresponding graph by considering end behavior and asymptotes.

1. $f(x) = \frac{(x+2)}{(2x+1)}$ III

2. $f(x) = \frac{(x^2+2)}{(2x+1)}$ I

3. $f(x) = \frac{(x+2)}{(2x^2+1)}$ IV

4. $f(x) = \frac{(x^3+2)}{(2x^2+1)}$ II



For each function find the following (if they exist).

- End Behavior including the equations of horizontal or slant asymptotes
- Vertical Asymptote(s). Distinguish between VA and Holes.

5. $f(x) = \frac{2x-1}{6x+3}$ $\frac{2x-1}{3(2x+1)}$
 EB = $\frac{2x}{6x} = \frac{1}{3}$ $y = \frac{1}{3} = \text{HA}$
 V.A. $x = -\frac{1}{2}$ V.A.

6. $f(x) = \frac{2}{x^3-x}$ $\frac{2}{x(x^2-1)} = \frac{2}{x(x+1)(x-1)}$
 EB: $\frac{2}{x^3}$ $y = 0$ HA
 V.A. $x = 0$
 $x = 1$ all V.A.
 $x = -1$

7. $f(x) = \frac{2x^2+5x+2}{x^2-1}$ $\frac{(2x+1)(x+2)}{(x+1)(x-1)}$
 EB: $\frac{2x^2}{x^2} = 2$ $y = 2$ HA
 V.A. $x = -1$ } all V.A.
 $x = 1$ }

8. $f(x) = \frac{x^2-2x+3}{x+2}$ *not factorable*
 EB = $\frac{x^2}{x} \rightarrow \infty$
 Slant asymptote $y = x - 4$

$$\begin{array}{r} -2 \overline{) 1-2 \ 3} \\ \underline{-2 \ 0} \\ 1-4 \end{array}$$

 V.A. $x = -2$ V.A.

9. $f(x) = \frac{x-1}{x^2+2x-3}$ $\frac{(x-1)}{(x-1)(x+3)}$
 EB: $\frac{x}{x^2} \rightarrow 0$ $y = 0$ HA
 V.A. $x = 1 \rightarrow \text{hole}$
 $x = -3 \rightarrow \text{VA}$

10. $f(x) = \frac{-3x^3}{x^3-4x}$ $\frac{-3x^3}{x(x^2-4)}$ $\frac{-3x^3}{x(x-2)(x+2)}$
 EB: $\frac{-3x^3}{x^3} = -3$ $y = -3$ HA
 V.A. $x = 2$ all V.A.
 $x = -2$ }

When a rational function has a **linear** polynomial divided by a **linear** polynomial, we have a special rational function that makes the Inverse Linear parent function $f(x) = \frac{1}{x}$. We use long division rewrite a rational function into the Inverse Linear form. Consider

the function $k(x) = \frac{3x+1}{x-1}$...we divide to get...

$$x-1 \overline{) \frac{3x+1}{x-1}} \quad \frac{3}{x-1} \quad \text{or } k(x) = \frac{4}{x-1} + 3.$$

$$\begin{array}{r} 3 \\ x-1 \overline{) 3x+1} \\ \underline{-(3x-3)} \\ 4 \end{array}$$

So, $k(x)$ is simply $f(x) = \frac{1}{x}$ with a vertical stretch of 4, translated right 1 and up 3. We can now easily graph the function.

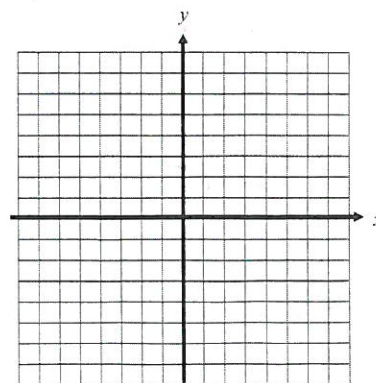
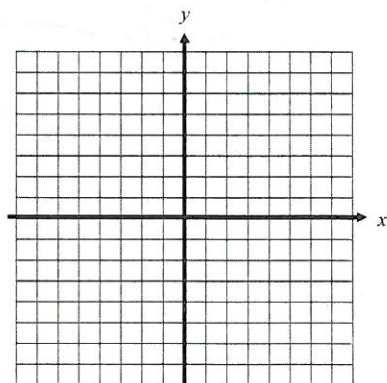
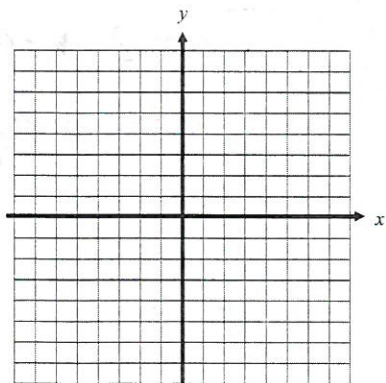
11. After reading the information above, graph $k(x)$.

For 13 – 15, use long division to obtain a function whose parent is $f(x) = \frac{1}{x}$. Describe the transformation and graph the function. Be sure to list the HA and VA.

13. $f(x) = \frac{2x-1}{x+3}$

14. $g(x) = \frac{3x-2}{x-1}$

15. $h(x) = \frac{5-2x}{x+4}$



Pre Calculus

Worksheet 2.7 Day 2

#15

For each function find the following (if they exist).

- End Behavior including the equations of horizontal or slant asymptotes ... Write as an equation of a line.
- Vertical Asymptote(s) ... Write as an equation of a line.
 - Hole(s) ... Write as an ordered pair.
- x- intercept(s) ... Write as an ordered pair.
- y- intercept ... Write as an ordered pair.
- Graph without a calculator. Identify *enough* additional points in each "region" to determine shape of graph.

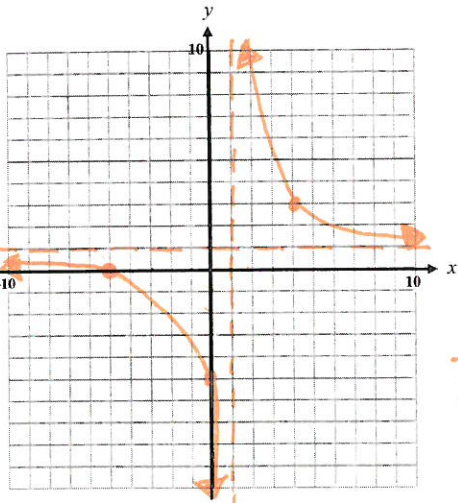
1. $f(x) = \frac{x+5}{x-1}$

EB: $\frac{x}{x} y = 1$
HA = 1

VA: $x = 1$

x-int (numer=0)

$x+5=0$
 $x=-5$
(-5, 0)



y-int (x=0)
 $\frac{0+5}{0-1} = -5$ (0, -5)

other pt(s) $x=4$ (4, 3)
 $\frac{4+5}{4-1} = \frac{9}{3} = 3$

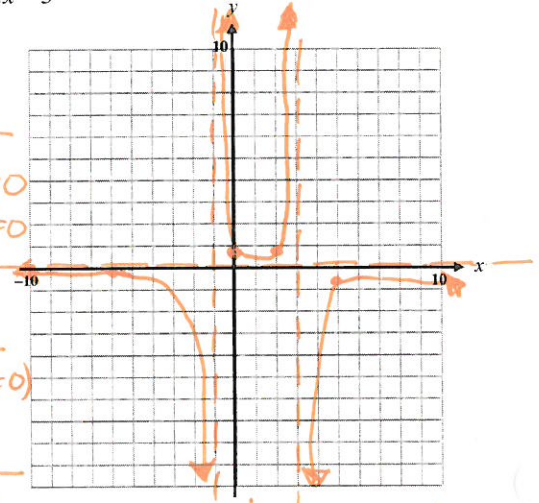
2. $f(x) = \frac{-2}{x^2-2x-3}$

EB: $y = 0$
HA = 0

VA: $x^2-2x-3=0$
 $(x-3)(x+1)=0$

$x=3$ $x=-1$

x-int (numer=0)
none



y-int
 $x=0$ $\frac{-2}{0^2-2(0)-3} = \frac{2}{3}$
 $(0, \frac{2}{3})$

other pts
 $x=-5 \rightarrow \frac{-2}{32} = -\frac{1}{16}$
 $x=2 \rightarrow \frac{-2}{-3} = \frac{2}{3}$
 $x=5 \rightarrow \frac{-2}{12} = -\frac{1}{6}$

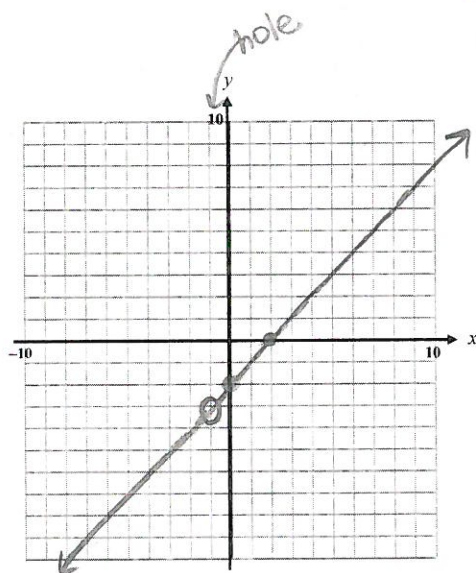
3. $f(x) = \frac{(x+1)(x-2)}{x^2-x-2} - 1$
 $\frac{x^2-x-2}{x^2-x-2} - 1$

eb $\frac{x^2}{x} \pm \infty$ slant asymptote $x-2$

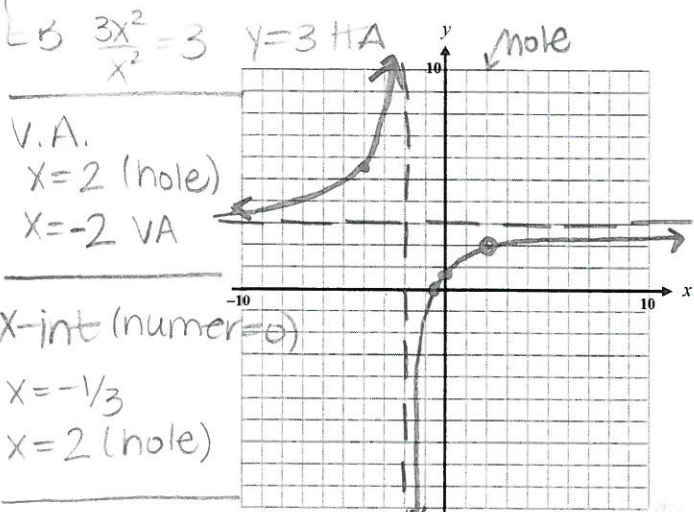
VA: $x = -1$ (hole) (-1, -3)

x-int (numer=0)
 $x = -1$ (hole) $x = 2$

y-int (x=0) $\frac{-2}{1} = -2$



$$4. f(x) = \frac{3x^2 - 5x - 2}{x^2 - 4} = \frac{(3x+1)(x-2)}{(x-2)(x+2)}$$



x-int (numerator=0)

$x = -1/3$
 $x = 2$ (hole)

y-int ($x=0$)

$\frac{-2}{-4}$

other pts ($x=-4$)

$$\frac{3(-4)^2 - 5(-4) - 2}{(-4)^2 - 4} = \frac{66}{12} = 5.5$$

$$5. f(x) = \frac{x+1}{x^2 - 3x - 10} = \frac{x+1}{(x-5)(x+2)}$$

EB $\frac{x}{x^2} = 0$ $y=0$ HA

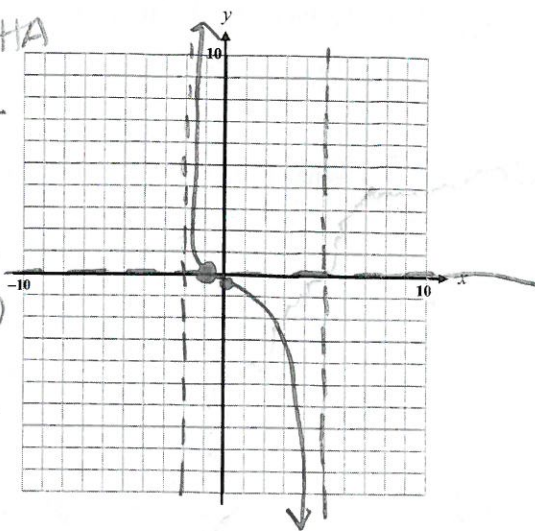
VA $x=5$ VA
 $x=2$ VA

x-int (numerator=0)

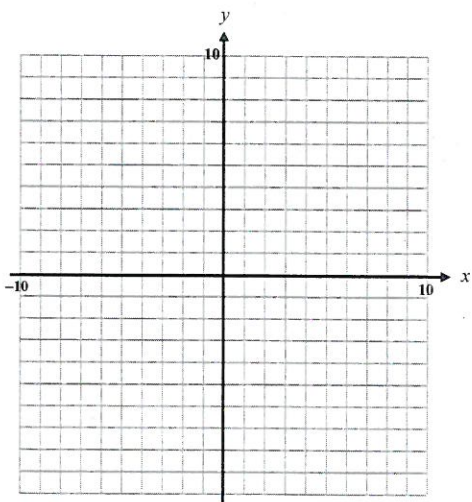
$x = -1$

y-int ($x=0$)

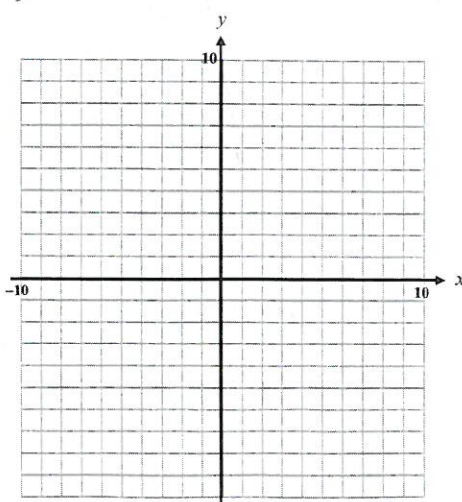
$\frac{1}{-10} = -\frac{1}{10}$



~~6.~~ $f(x) = \frac{x^2 - 3x - 4}{x - 3}$



~~7.~~ $f(x) = \frac{2x^2 + 5x + 3}{x^2 - 9}$



#1-4, first #5, 8, 9

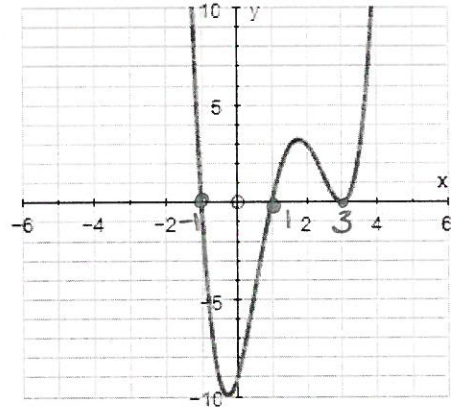
1. Use the graph of $g(x)$ at the right to answer each question.

a) When is $g(x) > 0$? $(-\infty, -1) \cup (1, 3) \cup (3, \infty)$

b) When is $g(x) \geq 0$? $(-\infty, -1] \cup [1, \infty)$

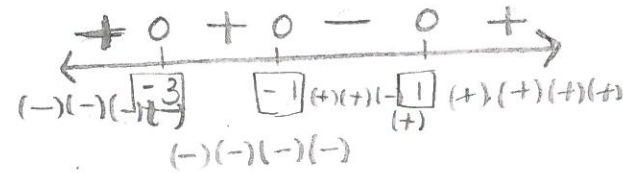
c) When is $g(x) < 0$? $(-1, 1) \cup (3, \infty)$

d) When is $g(x) \leq 0$? $[-1, 1] \cup [3, \infty)$



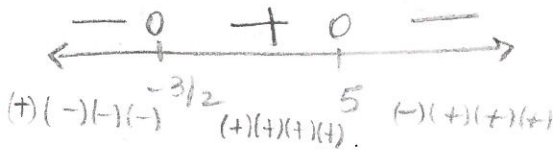
2. Solve the following inequality algebraically: $(x+3)^2(x-1)(x+1) < 0$.

$(-1, 1)$



3. Compare your answer from question 2 to question 1c. What do you notice? *MUST be same equation.*

4. Solve the following inequality using a graph AND algebraically: $(5-x)(2x+3) \geq 0$



$x = 5$ $x = -3/2$

$[-3/2, 5]$

5. Consider the function $f(x) = \frac{(2x)(x-4)}{(x+7)(3x-5)}$.

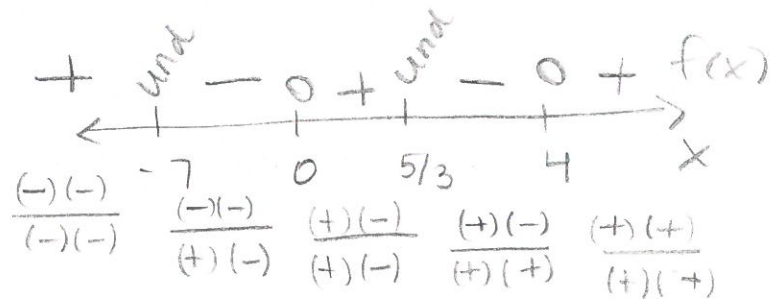
a) When does $f(x) = 0$? $x = 0$

$x = 4$

b) When is $f(x)$ undefined?

$x = -7$

$x = 5/3$



c) Create a sign chart to determine when $f(x) \geq 0$.

$f(x) \geq 0$ when $(-\infty, -7) \cup [0, 5/3) \cup [4, \infty)$

d) Use the sign chart you created in part c to determine when $f(x) < 0$.

$f(x) < 0$ when $(-7, 0) \cup (5/3, 4)$

5. Solve each inequality by first completing factoring the left side and creating a sign chart.

a) $x^3 - 4x^2 - x + 4 > 0$

b) $\frac{x^2 + 3x - 10}{x^2 - 6x + 9} < 0$

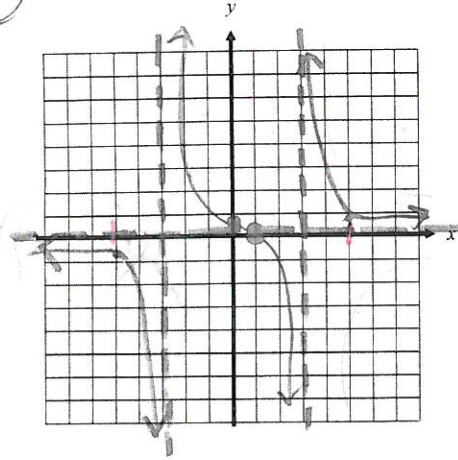
6. A sign chart in question 5b involved a rational function that was less than zero. You may first need to add or subtract fractions in order to create a rational function. You may also need to make one side equal 0 before using a sign chart to solve. Solve the following problems using a sign chart.

a) $\frac{1}{x+2} - \frac{2}{x-1} > 0$

b) $\frac{4x+5}{x+2} \geq 3$

In lesson 2.7 we graphed rational functions by first finding the key attributes of rational functions.

8. Find the key attributes (all asymptotes, intercepts, and holes) and create a graph to solve the inequality: $\frac{x-1}{x^2-9} \leq 0$



$$y = \frac{x-1}{(x-3)(x+3)} \leq 0$$

HA $y=0$

VA $x=3, x=-3$

X-int (numerator=0)

$x=1$

Y-int (plug $x=0$)

$\frac{-1}{3(-3)} = \frac{-1}{-9} = \frac{1}{9}$

$\frac{-1}{-9} = \frac{1}{9}$

$\frac{4}{16} = \frac{1}{4}$

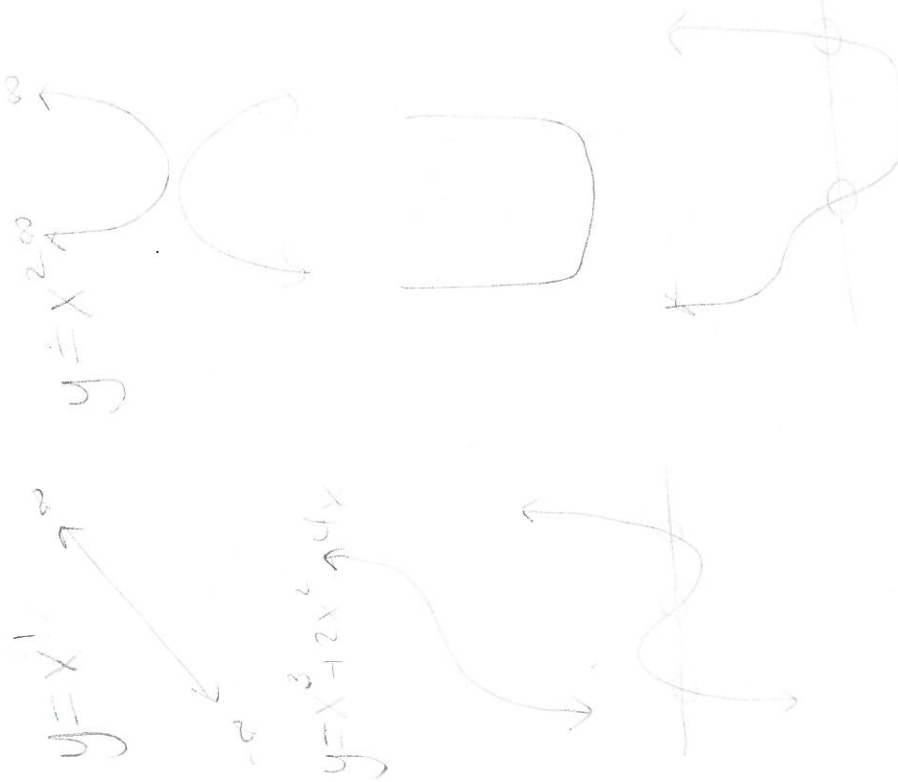
other pts

$x = -5 = \frac{-6}{16}$

$x = -1$

$x = 5$

9. Using what you know about sign charts, solve the inequality above. Could making a sign chart help you create graphs of rational functions? Explain how.



Need more practice? Ask your teacher to check out a textbook.

Like #2 and #3: Already Factored Polynomial Inequalities: page 265 #7 and 8

Like #5a and #5b: Factoring Required Polynomial Inequalities: page 265 #9 - 20

Like #4 and #5c: Rational Inequalities (with or without factoring): page 266 #25, 26 - 40

Like #6 and #7: Write the Rational Inequality as a Single Fraction First: page 266 #47, 49