KEY

Precalculus — PRE-CALC —
Worksheet P.2

1. What is the distance formula? Use it to find the distance between the points (5, 19) and (–3, 7).

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\sqrt{(-3-5)^2+(7-19)^2}$

$\sqrt{(-8)^2+(-12)^2}$

$\sqrt{64+144}$

$\boxed{\sqrt{208}=4\sqrt{13}}$

104 ② , ② 52, ② 26, ② 13

2. What is the midpoint formula? Use it to find the midpoint between the points (5, 19) and (–3, 7).

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\left(\frac{5+-3}{2}, \frac{19+7}{2}\right)$

$\frac{2}{2}, \frac{12}{2}$ $\boxed{(1, 13)}$

(1, 13)

3. Suppose the center of the circle is (4, –3) and another point on the circle at (–6, 2). Write the equation of the circle.

radius = distance between (4, -3) and (-6, 2)

$r=\sqrt{(-6-4)^2+(2+{+3})^2}$

$=\sqrt{(-10)^2+(5)^2}$

$=\sqrt{125}$

radius $=\sqrt{125}$

center = (4, -3)

$\boxed{125=(x-4)^2+(y+3)^2}$

r^2

4. Consider the equation $(x+8)^2+y^2=13$.

a) What is the center and radius of the circle?

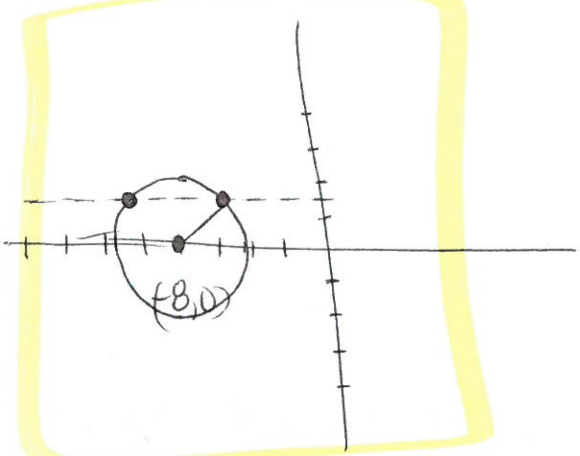
center (-8, 0)

radius $\sqrt{13}\approx 3.5$

b) Graph the circle. Find all points on the circle that have a y-value of 3.

$(x+8)^2+(y-0)^2=13$

3



y-value

$(x+8)^2+(3-0)^2=13$

$(x+8)^2+9=13$

$\sqrt{(x+8)^2}=\sqrt{4}$

$x+8=\pm 2$

$x+8=2$ $\quad$ $x+8=-2$

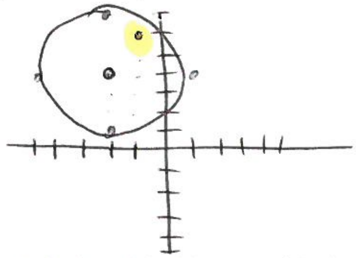
$x=-6$ $\quad$ $x=-10$

5. Consider the equation of the circle $(x+2)^2+(y-4)^2=9$.

a) What is the center and radius of the circle?

center $(-2,4)$ radius $=3$

b) Graph the circle by hand.



c) Is the point (–1, 6) inside or outside the circle? PROVE it algebraically.

$(-1+2)^2+(6-4)^2=$

$1^2+2^2=5$

$\sqrt{5}=$ radius

radius is smaller than 3 so it's inside circle

d) Use your graphing calculator to graph the circle. Explain/Show what you must do in order to make this possible.

$(x+2)^2+(y-4)^2=9$

$\sqrt{(y-4)^2}\pm=\sqrt{9-(x+2)^2}$

$y-4=\sqrt{9-(x+2)^2}+4$

$+4$

6. Suppose the endpoints of the diameter of a circle are (5, 19) and (–3, 7). Write the equation of the circle.

Midpoint = center

$\left(\frac{5+-3}{2}, \frac{19+7}{2}\right)=\boxed{(1,13)}$

radius $=\sqrt{52}$

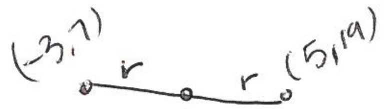
radius = distance between center and point on circle

$=\sqrt{(5-1)^2+(19-13)^2}$

$=\sqrt{4^2+6^2}$

$=\sqrt{16+36}$

$=\sqrt{52}$



$r^2=(x-h)^2+(y-k)^2$

$\boxed{52=(x-1)^2+(y-13)^2}$

Precalculus
Worksheet P.1

1-13 skip 7

1. Describe in words and graph each interval of real numbers.

a) $x \le 2$

all values less than or equal to 2

1 2 3

b) $-2 \le x < 5$

all values greater than or equal to −2 but less than 5

−2 5

c) $(-\infty, 7)$

all values less than 7

7

d) $[-3, 3]$

all values between and including −3 and 3

−3 3

2. Graph the interval described in words and use interval notation to describe the interval.

a) x is negative

0

$(-\infty, 0)$

b) x is greater than or equal to 2 and less than or equal to 6

2 6

$[2, 6]$

3. Explain why each c ... ten:

a) $5 < x > 7$

If we wanted only numbers greater than 5 & 7, we could just write $x > 7$.

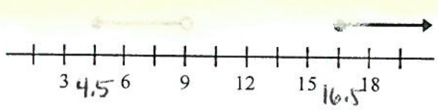
...r ... han

b) $3 > x > 8$

< <

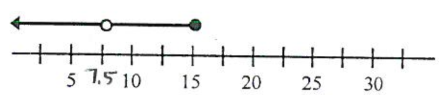
It is not possible for a number to be less than 3 and greater than 8

4. D ... rval notation.



Ineq: $4.5 \le x < 9$ or $x \ge 16.5$

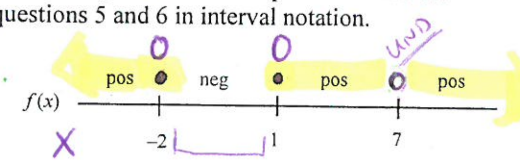
interval $[4.5, 9) \cup [16.5, \infty)$



Ineq: $x < 7.5$ or $7.5 < x \le 15$

interval: $(-\infty, 7.5) \cup (7.5, 15]$

Later in the course we will be using sign charts. Sign charts are simply number lines with "pos" or "neg" signs on them to represent whether or not the function or equation is positive or negative on that interval. Suppose the sign chart below represents when the function $f(x)$ is positive "pos" or negative "neg". Use the sign chart to answer the questions 5 and 6 in interval notation.



5. When is $f(x) < 0$? … (i.e. when is the function negative?)

$(-2, 1)$

6. If $f(x) = 0$ when $x = -2$ and $x = 1$, but not when $x = 7$, when is $f(x) \ge 0$?

$(-\infty, -2] \cup [1, 7) \cup (7, \infty)$

7. Use both inequality AND interval notation to describe the set of numbers. ***Define any variables used***.

a) In Excel, you can create an if-then statement that will enter a specific value or text into a box depending on given conditions. Suppose you would like to enter a value of "YES" into cell A8 if the value in cell A7 is between 4 and 16 including both.

b) A programmer wants to create an alertbox if the user inputs a value that is at least 150.

$-(4)(4)$ $\quad (-4)(-4)$

8. Identify the base in each of the following expressions, then evaluate each expression.

a) -4^2 b) $(-4)^2$ c) $(-3)^3$ d) -3^3 e) $2x+1^2$ f) $(2x+1)^2$

a)	b)	c)	d)	e)	f)
base 4	base -4	base -3	base 3	base 1	base $2x+1$
-16	16	-27	-27	$2x+1$	$(2x+1)(2x+1)$ $4x^2+4x+1$

9. Simplify each of the following expressions. Assume that the variables in the denominators are nonzero.

a) $\dfrac{x^4y^3}{x^2y^5} = \boxed{\dfrac{x^2}{y^2}}$

b) $\dfrac{(3x^2)^2y^4}{3y^2}$

$= \dfrac{9x^4y^4}{3y^2} = \boxed{3x^4y^2}$

c) $\left(\dfrac{4}{x^2}\right)^2 = \boxed{\dfrac{16}{x^4}}$

d) $\left(\dfrac{2}{xy}\right)^{-3} = \left(\dfrac{xy}{2}\right)^3$

$= \boxed{\dfrac{x^3y^3}{8}}$

e) $\dfrac{(x^{-3}y^2)^{-4}}{(y^6x^{-4})^{-2}}$

$\dfrac{x^{12}y^{-8}}{y^{-12}x^8} = \boxed{x^4y^4}$

f) $\left(\dfrac{4a^3b}{a^2b^3}\right)\left(\dfrac{3b^2}{2a^2b^4}\right)$

$= \dfrac{12a^3b^4}{2a^4b^7} = \boxed{\dfrac{6}{ab^4}}$

10. Match the following equations with the property illustrated. Each letter is only used once.

D $(3x)y = 3(xy)$

B $a^2b = ba^2$

G $a^2b + (-a^2b) = 0$

E $(x+3)^2 + 0 = (x+3)^2$

I $a(x+y) = ax + ay$

H $(x+2)\cdot\dfrac{1}{(x+2)} = 1$

F $1\cdot(x+y) = x+y$

A $(a+b)+c = c+(a+b)$

C $x^2y + (z+a^3) = (x^2y+z)+a^3$

A) Commutative Property of Addition

B) Commutative Property of Multiplication

C) Associative Property of Addition

D) Associative Property of Multiplication

E) Identity Property of Addition

F) Identity Property of Multiplication

G) Inverse Property of Addition

H) Inverse Property of Multiplication

I) Distributive Property

11. Simplify each expression.

a) $(4xy+2x^3)ax^2$

$4ax^3y + 2ax^5$

b) $3a(2a+5b)$

$6a^2 + 15ab$

Many times we need to undo the distributive property. This is the beginning of factoring. When you undo the distributive property, we say you have ***factored out the greatest common factor.***

12. Factor out the GCF from each expression.

a) $-3x^2-6xy$

$-3x(x+2y)$

b) $108x^3-36x^2+60x$

$12x(9x^2-3x+5)$

c) ☆²♫³👽⁶ + ☆♫⁵👽²

☆♫³😐²(☆😐⁴ + ♫²)

13. Describe in your own words what the phrase "Take out a GCF" means.

it means to divide out the largest factor that all the terms have in common.

$\boxed{\#1\text{-}8 \text{ skip } 3}$ 1, 2, 6-8

Precalculus
Worksheet P.3

1. Solve each of the following equations algebraically:

a) $4(y-2)=5y$

$4y-8=5y$

$-8=y$

b) $2\left(\frac{1}{2}x=\frac{7}{8}\right)2$

$x=\frac{14}{8}$

$x=\frac{7}{4}$

6 c) $\left(\frac{1}{2}x+\frac{1}{3}=1\right)$

$3x+2=6$

$3x=4$

$x=4/3$

4 d) $\left(\frac{2x-3}{4}+5=3x\right)$

$2x-3+20=12x$

$17=10x$

$\frac{17}{10}=x$

24 e) $\left(\frac{t+5}{8}-\frac{t-2}{2}=\frac{1}{3}\right)$

$3(t+5)-12(t-2)=8$

$3t+15-12t+24=8$

$-9t+39=8$
$\quad\;\; -39 \quad -39$

$-9t=-31$

$t=+31/9$

2. The formula for the perimeter of a rectangle is given by $P = 2L + 2W$. Solve this equation for W.

$W=\frac{P-2L}{2}$

X 3. The formula for the area of a trapezoid is given by $A=\frac{1}{2}h(b_1+b_2)$. Solve this equation for b_1.

$A=\frac{1}{2}h(b_1+b_2)$

$2A=h(b_1+b_2)$

$\frac{2A}{h}=b_1+b_2$

$\boxed{\frac{2A}{h}-b_2=b_1}$

4. A formula relating Fahrenheit to Celsius is $C=\frac{5}{9}(F-32)$. Solve this equation for F.

$F=\frac{9}{5}C+32$

5. A satellite orbiting the planet has a mass m, a velocity v, and a radius from the center of the planet equal to R. The centrifugal Force F acting on that satellite is given by the equation $F=\frac{mv^2}{R}$. Solve this equation for R.

$R=\frac{mv^2}{F}$

6. You have scores of 68, 82, 87, and 89 on your first four tests, and you have one more to take. In order to get a B you must have an average between 79.5 and 89.5. What is the lowest score you could get on your last test in order to get a B.

$$\frac{68+82+87+89+x}{5} \geq 79.5 \qquad 326+x \geq 397.5$$

$$x \geq 71.5$$

7. When you solve an inequality, when does the inequality change direction. *Explain why*.

8. Solve each of the following inequalities and write your answer in interval notation.

a) $2 \leq x+6 < 9$

$-4 \leq x < 3$

$[-4, 3)$

b) $-1 \leq 3x-2 < 7$

$1 \leq 3x < 9$

$\frac{1}{3} \leq \cancel{x} < 3$

$[1/3, 3)$

c) $4\left(\frac{5x+7}{4} \leq -3\right)$

$5x+7 \leq -12$

$5x \leq -19$

$x \leq -19/5$

$(-\infty, -19/5]$

d) $3\left(4 \geq \frac{2y-5}{3} \geq -2\right)$

$12 \geq 2y-5 \geq -6$

$17 \geq 2y \geq -1$

$\frac{17}{2} \geq y \geq -\frac{1}{2}$

$-\frac{1}{2} \leq y \leq 17/2$

$[-\frac{1}{2}, 17/2]$

e) $24\left(\frac{3-4y}{6} - \frac{2y-3}{8} \geq 2-y\right)$

$4(3-4y) - 3(2y-3) \geq 48-24y$

$12-16y-6y+9 \geq 48-24y$

$21-22y \geq -24y+48$

$2y \geq 27$

$y \geq 27/2$

$[27/2, \infty)$

f) $6\left(\frac{1}{2}(x+3)+2(x-4) < \frac{1}{3}(x-3)\right)$

$3x+9+12x-48 < 2x-6$

$15x-39 < 2x-6$

$13x < 33$

$x < 33/13$

$(-\infty, 33/13)$

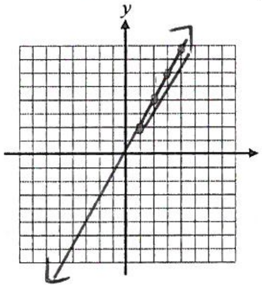
P.4 #1,2

Precalculus
Worksheet P.4

1. So far, we have covered 5 types of linear equations: Point-Slope, Slope-Intercept, Standard, Horizontal, & Vertical. Identify each form by name and graph each linear equation on the grids provided.

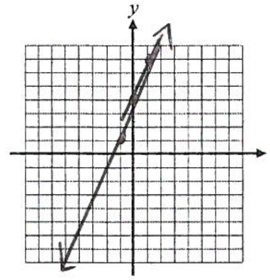
a) $y-2=3(x-1)$ (1,2)

Name: point-slope



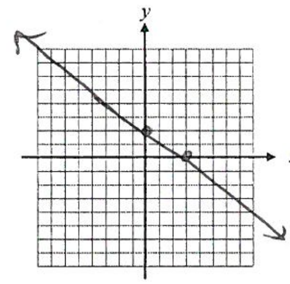
b) $y=3x+4$

Name: Slope int.



c) $2x+3y-6=0$ $2x+3y=6$

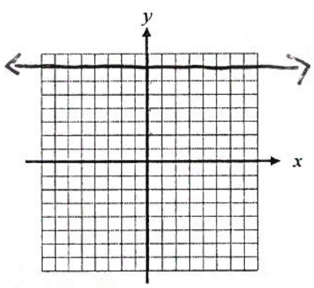
Name: Standard/generae



X	y
0	2
3	0

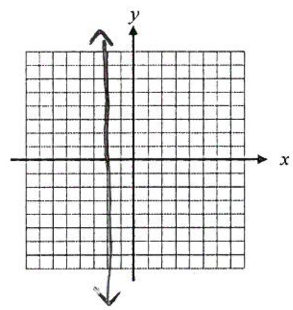
d) $y=7$

Name: horizontal



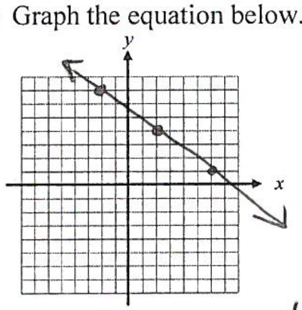
e) $x=-2$

Name: vertical



2. Consider the line that passes through the point (2, 4) and has a slope of $-\frac{3}{4}$.

a) Graph the equation below.



b) Write the equation of the line in point-slope form.

$\boxed{y-4=-3/4(x-2)}$

c) Write the equation of the line in slope-intercept form.

$y=-3/4x+3/2+4$

$\boxed{y=-3/4x+} \ 11/2$

d) Write the equation of the line in general form.

$4y=-3x+22$

$\boxed{3x+4y=22}$

e) Write the equation of the line that is parallel to this line and goes through the point (–9, 5).

$y-5=-3/4(x+9)$

f) Write the equation of the line that is perpendicular to this line and goes through the point (–9, 5).

$y-5=4/3(x+9)$

3. Remember, we want to think of slope as a rate of change.

4. On a recent trip, you noticed that after 2 hours you were 90 miles from home, but after 5 hours you were 270 miles from home.

hours miles

(2, 90) (5, 270)

a) Find the average rate of change from your first to your second observation.

$$\frac{270-90}{5-2} = \frac{180}{3} = 60 \qquad \frac{\Delta y}{\Delta x} = \frac{\text{miles}}{\text{hrs}}$$

b) What are the units of measurement? What does the average rate of change in your position tell you?

$\frac{\text{miles}}{\text{hour}}$ avg rate of change of position gives you speed.

5. The velocity of a rocket launched when $t = 0$ was recorded for selected values of t over the interval $0 \leq t \leq 80$. Acceleration is the rate of change in velocity (a.k.a. slope of velocity). Finding average acceleration between two points can be done by finding the slope between those two points on the velocity function.

t (seconds)	0	10	20	30	40	50	60	70	80
Velocity (feet/second)	5	14	22	29	35	40	44	47	49

Find the average acceleration of the rocket from $t = 0$ to $t = 80$. What are the units of measurement?

(0,5) (80,49)

$$\frac{49-5}{80-0} = \frac{44}{80} = \frac{11}{20} \approx .55 \text{ feet/sec}$$

6. Find the value of x or y so that the line through the pair of points has the ~~given~~ slope.

a) Points $(x, 2)$ and $(4, 8)$ with slope = 2.

$$\frac{8-2}{4-x} = 2 \qquad \frac{6}{4-x} = 2 \qquad \boxed{x=1}$$

b) Points $(-1, 3)$ and $(4, y)$ with slope = ½ .

$$\frac{y-3}{4+1} = \frac{1}{2} \qquad \frac{y-3}{5} = \frac{1}{2} \qquad \boxed{y = 5.5}$$

7. A vertical asymptote occurs in a rational function when the denominator of that function is equal to zero. Consider the function $y = \frac{5}{x-3}$. What is the equation of the vertical asymptote? (Remember … it's a vertical line!)

$\boxed{x=3}$

8. A horizontal asymptote is the y-value that a function approaches when x gets really, really large (a.k.a. infinity). Graph the function $y = \frac{6x}{3x-7}$. Just by looking at the graph, what do you think is the equation of the horizontal asymptote? (Remember … it's a horizontal line)

$\boxed{y=2}$

9. Consider the equation $\frac{1}{3}x+2y=5$.

$\boxed{x+6y=15}$

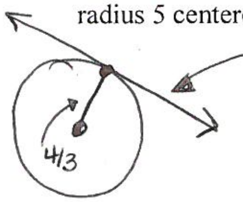
a) Explain why this equation is NOT in general form, and then "fix" it so that the equation is in general form.

all numbers must be integers

b) What is the slope of this line?

$-1/6$

10. A line that is tangent to a circle at a point is perpendicular to the radius at that point. Consider the circle of radius 5 centered at (0, 0). Find an equation of the line tangent to the circle at (3, 4).



slope $-\frac{3}{4}$

$y-4=-\frac{3}{4}(x+3)$

11. The relationship between Fahrenheit and Celsius temperatures is linear.

a) Use the facts that water freezes at 0° C or 32° F, and water boils at 100° C or 212° F (not your recollection of temperature formulas) to find an equation that relates Celsius and Fahrenheit.

(0, 32) (100, 212)

$\frac{212-32}{100-0}=\frac{180}{100}=1.8=\frac{9}{5}$

$y=\frac{9}{5}x+32$

$F=9/5C+32$

b) Using your equation, find the Fahrenheit equivalent of –10° C.

$F=9/5(-10)+32$

$F=14°$

12. Consider the function $y=2x^2-7$. The tangent line to this curve at the point (–3, 11) has a slope of –12. This line is called a tangent line to the curve, because it shares a common point and slope at that point.

a) Find the equation of the tangent line in slope-intercept form.
[Graph both the curve and the tangent line on the same screen on your calculator]

$y-11=-12(x+3)$

$y=-12x+25$

b) In Calculus, we will use the tangent line to approximate the value of the function. (in other words … we find the y-value on the line instead of the y-value on the curve) Use the tangent line to approximate the value of the function when $x=-1$.

line $x=-1$ $\quad y=-12(-1)-25$ $\quad y=-13$ $\quad \boxed{(-1,-13)}$

$= 12-25$

c) Your answer from part b is called a linear approximation. The error is the difference between the estimated value using the tangent line and the actual value from the curve. Find the error in your approximation.

$(-1,-5) \rightarrow (-1,-13)$

error is -8

Precalculus
Worksheet Factoring

#1-3

1. Learning to recognize the difference of two perfect squares is more difficult than factoring them. Factor the following expressions, and write something on this paper that will help you recognize this in the future.

a) $a^2 - 25$

$(a+5)(a-5)$

b) $16 - n^2$

$(4-n)(4+n)$

2. Factor the following quadratic expressions completely:

a) $b^2 + 8b + 15$

$(b+5)(b+3)$

b) $6d^2 + 35d - 6$

$(6d-1)(d+6)$

c) $10t^2 - 19t + 6$

$(2t-3)(5t-2)$

d) $p^3 - 4p^2 - 5p$

$p(p^2-4p-5)$

$p(p-5)(p+1)$

e) $12f^3 - 14f^2 - 40f$

$2f(6f^2-7f-20)$

$2f(2f-5)(3f+4)$

f) $20e^4 - 23e^3 + 6e^2$

$e^2(20e^2-23e+6)$

$e^2(4e-3)(5e-2)$

3. Factor the following equations completely using the grouping method:

a) $ax + ay + 6x + 6y$

$a(x+y) \mid 6(x+y)$

$(a+6)(x+y)$

b) $mp^2 + 7m + 3p^2 + 21$

$m(p^2+7) \mid 3(p^2+7)$

$(m+3)(p^2+7)$

c) $b^3 - 7b^2 - 9b + 63$

$b^2(b-7) \mid -9(b-7)$

$(b^2-9)(b-7)$ * $(b-3)(b+3)(b-7)$

4. Factor each of the following expressions:

a) $6(5x-3)^2 - 11(5x-3) - 7$

$6(s)^2 - 11(s) - 7$

$(2s+1)(3s-7)$

$(2(5x-3)+1)(3(5x-3)-7)$

$(10x-6+1)(15x-9-7)$

$\boxed{(10x-5)(15x-16)}$

b) $5(8x+9)^2 + 47(8x+9) + 18$

$5y^2 + 47y + 18$

$(5y+2)(y+9)$

$(5(8x+9)+2)((8x+9)+9)$

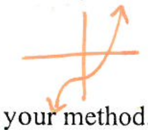
$(40x+45+2)(8x+18)$

$\boxed{(40x+47)(8x+18)}$

Precalculus
Worksheet P.5

The main purpose of section P.5 is for you to be able to solve quadratic functions in the most efficient (quickest) method possible. In order to do this you should learn as many different methods as possible.

1. Graphically solve the equation $x^3+x^2+2x-3=0$. Describe your method.

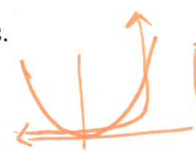
Calculate → zero  $\boxed{x=.844}$ ← round to 3 decimal places!

2. Graphically solve the equation $x^2+4=4x$. Describe your method.

$\left.\begin{array}{l} y_1=x^2+4 \\ y_2=4x \end{array}\right\}$ calculate intersection  $\boxed{x=2}$

3. Solve the equation $2^x=x^2$ graphically two different ways … keep in mind you should get the same answer(s).

a) Solve by finding the intersections.

$\left.\begin{array}{l} y_1=2^x \\ y_2=x^2 \end{array}\right\}$ Calc → intersec.  $\boxed{x=-.766, 2, 4}$

b) Solve by finding the zeros.

$y=2^x-x^2$ $\boxed{x=-.766, 2, 4}$ 

4. The table below shows the *x*- and *y*-coordinates of the equation $y=x^2+2x-1$. State the expression and give the zero as accurately as can be read from the table.

$0=x^2+2x-1$

the zero is between 0.41 & 0.42

x	y
0.4	-0.04
0.41	-0.0119
0.42	0.0164
0.43	0.0449
0.44	0.0736
0.45	0.1025
0.46	0.1316

5. Solve each of the following equations by using factoring:

a) $4x^2-8x+3=0$

	$2x$	-1
-3	$-6x$	3
$2x$	$4x^2$	$-2x$

$12x^2$; $-2x$, $-6x$; $-8x$

$(2x-3)(2x-1)=0$

$\boxed{x=3/2 \quad x=1/2}$

b) $x(3x+11)=20$

$3x^2+11x=20$

$3x^2+11x-20=0$

	$3x$	-4
5	$15x$	-20
x	$3x^2$	$-4x$

$-60x^2$; $-4x$, $15x$; $+11x$

$(x+5)(3x-4)=0$

$\boxed{x=-5, x=4/3}$

c) $x^3+x^2-9x-9=0$

$x^2(x+1)-9(x+1)$

$(x^2-9)(x+1)=0$

$(x+3)(x-3)(x+1)=0$

$\boxed{x=-3 \quad x=3 \quad x=-1}$

6. Solve each of the following equations by "extracting square roots".

a) $3x^2-5=43$

$3x^2=48$

$x^2=16$

$x=\sqrt{16}$

$\boxed{x=\pm 4}$

b) $3(x+5)^2=27$

$(x+5)^2=9$

$x+5=\sqrt{9}$

$x+5=\pm 3$

$x+5=3$ $\quad$ $x+5=-3$

$\boxed{x=2}$ $\quad$ $\boxed{x=-8}$

c) $\frac{1}{2}2\left(x-\frac{3}{4}\right)^2=\frac{49}{32}\cdot\frac{1}{2}$

$\left(x-\frac{3}{4}\right)^2=\frac{49}{64}$

$\left(x-\frac{3}{4}\right)=\sqrt{\frac{49}{64}}$

$x-\frac{3}{4}=\frac{7}{8}+\frac{6}{8}$ $\quad$ $x-\frac{3}{4}=\frac{7}{8}+\frac{6}{8}$

$\boxed{x=1\tfrac{3}{8} \text{ or } x=-\tfrac{1}{8}}$

7. Solve each of the following equations by "completing the square"

a) $x^2-6x=7$

b) $3x^2+x-2=0$

c) $-2x^2+5x+1=0$

8. Solve the equation using the quadratic formula: $x(x+5)=12$

$x^2+5x=12$

$x^2+5x-12=0$

$a=1 \quad b=5 \quad c=-12$

$$\frac{-5\pm\sqrt{5^2-4(1)(-12)}}{2(1)}$$

$$\boxed{\frac{-5\pm\sqrt{73}}{2}}$$

$\frac{-5+\sqrt{73}}{2}=\boxed{1.772}$

or

$\frac{-5-\sqrt{73}}{2}=\boxed{-6.772}$

Keep in mind that for the purposes of quizzes and tests, there will be times that you will be asked to use a specific method. However, in general, you may use whatever method works for you and is appropriate.

9. Several of the World Cup '94 soccer matches were played in Stanford University's stadium in Menlo Park, California. The field is 30 yd longer that it is wide, and the area of the fied is 8800 yd^2. What are the dimensions of the field?

$w+30$

w $\quad$ $\boxed{A=8800}$

$w(w+30)=8800$

$w^2+30w=8800$

(method of your choice)

~~$w=-110$~~

$w=80$

$\boxed{\text{width}=80\text{ yd} \quad \text{length}=110\text{ yd}}$

$2x^2+10x-299=0$

10. John's paint crew knows from experience that its 18-ft ladder is particularly stable when the distance from the ground to the top of the ladder is 5 ft more than the distance from the building to the base of the ladder as shown in the figure. In this position, how far up the building does the ladder reach?

$x^2+(x+5)^2=18^2$

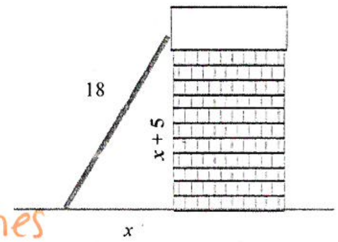
$x^2+x^2+10x+25=324$

$2x^2+10x+25=324$

(method of your choice)

$x=9.980$

$\cancel{x=-14.980}$



the ladder reaches $9.980+5=14.980$ ft up the building

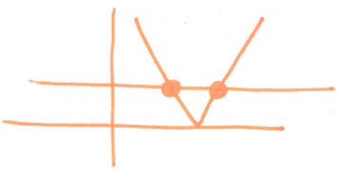
The second portion of this section is for you to solve equations involving absolute value.

11. Solve each of the following equations graphically AND algebraically:

a) $|t-8|=2$

$t-8=2$ $\quad$ $t-8=-2$

$\boxed{t=10 \text{ or } t=6}$



b) $5-2|3x+1|=-9$

$-2|3x+1|=-14$

$|3x+1|=7$

$3x+1=7$ $\quad$ $3x+1=-7$

$\boxed{x=2 \text{ or } x=-8/3}$

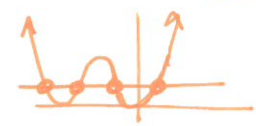
c) $|x^2+4x-1|=4$

$x^2+4x-1=4$ $\quad$ $x^2+4x-1=-4$

$x^2+4x-5=0$ $\quad$ $x^2+4x+3=0$

$(x+5)(x-1)=0$ $\quad$ $(x+1)(x+3)=0$

$\boxed{x=-5 \quad x=1}$ $\quad$ $\boxed{x=-1 \quad x=-3}$



11. The following equations represent things you should be able to do to this point. Solve each algebraically:

a) $-2(5-3x)+8=4+5x$

$-10+6x+8=4+5x$

$6x-2=4+5x$

$\boxed{x=6}$

b) $-3+5x^2=17$

$5x^2=20$

$x^2=4$

$\boxed{x=\pm 2}$

c) $\left(\frac{3x}{4}-\frac{x}{3}=\frac{1}{12}\right)\cdot 12$

$9x-4x=1$

$5x=1$

$\boxed{x=\frac{1}{5}}$

d) $|3x-1|=5$

$3x-1=5$ $\quad$ $3x-1=-5$

$3x=6$ $\quad$ $3x=-4$

$\boxed{x=2 \text{ or } x=-4/3}$

e) $x(2-x)=3(x-4)$

$2x-x^2=3x-12$

$0=x^2+x-12$

$0=(x+4)(x-3)$

$\boxed{x=-4,\ x=3}$

f) $4x^2=6x$

$4x^2-6x=0$

$2x(2x-3)=0$

$\boxed{x=0 \quad x=3/2}$

Precalculus
Worksheet P.6

#1-5, 6, 8-12

For questions 1 – 4, solve the inequality algebraically. Write your answer in interval notation.

1. $|x-8| \le 17$

$-17 \le x-8 \le 17$
$+8 \quad +8 \quad +8$
$-9 \le x \le 25$
$[-9, 25]$

2. $|3-2x| > 4$

$3-2x > 4$ $\quad -3 \quad -3$
$-2x > 1$
$\div -2 \quad \div -2$
$x < -\frac{1}{2}$

$3-2x < -4$ $\quad -3 \quad -3$
$-2x < -7$
$\div -2 \quad \div -2$
$x > \frac{7}{2}$

$(-\infty, -1/2) \cup (7/2, \infty)$

3. $3-2|x-9| \ge -9$

$-3 \quad -3$
$\frac{-2|x-9|}{-2} \ge \frac{-12}{-2}$
$|x-9| \le 6$
$-6 \le x-9 \le 6$
$+9 \quad +9 \quad +9$
$3 \le x \le 15$
$[3, 15]$

4. $5|2x-3|-8 < 13$

$+8 \quad +8$
$\frac{5|2x-3|}{5} < \frac{21}{5}$
$|2x-3| < \frac{21}{5}$
$-\frac{21}{5} < 2x-3 < \frac{21}{5}$
$+\frac{15}{5} \quad +3 \quad +\frac{15}{5}$
$\div 2 \quad -\frac{6}{5} < \frac{2x}{2} < \frac{36}{5} \quad \div 2$
$\frac{-6}{10} < x < \frac{36}{10}$
$\frac{-3}{5} < x < \frac{18}{5}$
$\left(-\frac{3}{5}, \frac{18}{5}\right)$

5. The $\sqrt{16} = 4$, but the equation $x^2 = 16$ has two solutions, $x = \pm 4$. The reason for this is the fact that $\sqrt{x^2} = |x|$. Using this fact, solve the following inequalities algebraically:

a) $x^2 - 9 > 0$

$+9 \quad +9$
$\sqrt{x^2} > \sqrt{9}$
$|x| > 3$
$-3 > x > 3$
$(-3, 3)$

b) $2(x-3)^2 + 5 < 13$

$-5 \quad -5$
$\frac{2(x-3)^2}{2} < \frac{8}{2}$
$\sqrt{(x-3)^2} < \sqrt{4}$
$|x-3| < 2$
$-2 < x-3 < 2$
$+3 \quad +3 \quad +3$
$1 < x < 5$
$(1, 5)$

6. Solve the following inequalities graphically. Write your answer in interval notation.

a) $2x^2 + 17x + 21 \le 0$

$[-7, -1.5]$

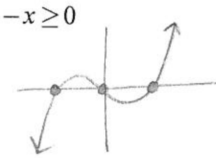
b) $2x^2 + 7x > 15$

$2x^2 + 7x - 15 > 0$
$x < -5$ or $x > 1.5$
$(-\infty, -5) \cup (1.5, \infty)$

c) $2 - 5x - 3x^2 > 8$

$-8 \quad -8$
$-6 - 5x - 3x^2 > 0$
NO SOLUTION

d) $x^3 - x \ge 0$



$[-1, 0] \cup [1, \infty]$

X

7. Solve the following inequalities graphically. Write your answer in interval notation.

a) $2x^3+2x>5$

b) $3x^3-12x+2<0$

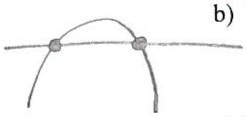
$h_0=0$ $V_0=272$

8. A projectile is launched straight up from ground level with an initial velocity of 272 ft/sec.

a) Using the equation for the height of a projectile, write an equation for the height (in feet) of an object as a function of time (in seconds).

$-16t^2+272t+0$

b) When will the projectile's height above the ground be 960 feet?



$t=5$ and $t=12$

c) When will the projectile's height above ground be more than 960 feet?

$(5,12)$

d) When will the projectile's height above ground be less than 960 feet?

$(0,5)\cup(12,17)$

e) What is the highest point the projectile will reach? When will it reach this height?

1156 ft at $t=8.5$ sec

9. For a certain gas, $P=400/V$, where P is the pressure and V is the volume. If $20\le V\le 40$, what is the corresponding range of P?

V	P
10	40
20	20
30	13.33
40	10
50	8

$P=\frac{400}{V}$

range: $10\le P\le 20$

10. The vertex of a parabola of the form $f(x)=ax^2+bx+c$ is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. Find the vertex of the function $g(x)=-2x^2+6x+7$ without using your calculator. Is this point a minimum or a maximum?

$\frac{-b}{2a}=\frac{-6}{2(-2)}=\frac{-6}{-4}=\frac{3}{2}$

$f(\frac{3}{2})=-2(\frac{3}{2})^2+6(\frac{3}{2})+7$

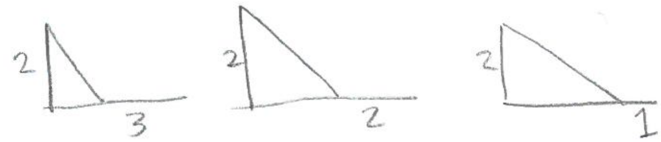
$=-2(\frac{9}{4})+9+7$

$=-\frac{9}{2}+9+7$

$=-\frac{9}{2}+16$

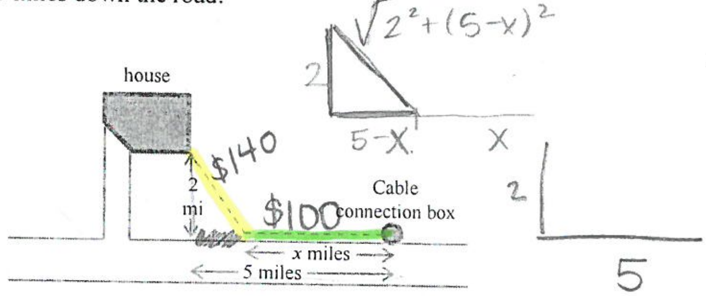
$-\frac{9}{2}+\frac{32}{2}=\frac{23}{2}$

$(\frac{3}{2},\frac{23}{2})$ is a maximum because $-2x^2+6x+7$ is a ∩ parabola



11. The cable company is asked to provide service to a customer whose house is located 2 miles from the road along which the cable is buried. The nearest connection box for the cable is located 5 miles down the road. (see figure below).

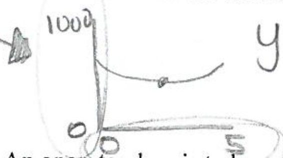
a) If the installation cost is $100 per mile along the road and $140 per mile off the road, express the total cost C of installation as a function of the distance x (in miles) from the connection box to the point where the cable installation turns off the road. What values of x make sense in this scenario?



cost along road = $100·x
cost off road = $140($\sqrt{4+(5-x)^2}$)

x can be any number $0 \leq x \leq 5$ [0,5]

b) What is the minimum possible cost of the installation, and how far away from the connection box should the cable installation turn off the road?

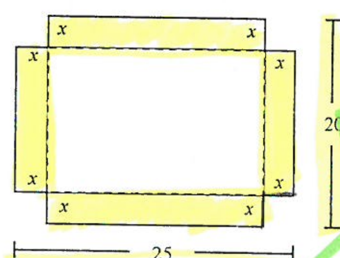


$y = 100x + 140\sqrt{4+(5-x)^2}$

minimum x=2.958 miles from connection box.

12. An open-top box is to be made by cutting congruent squares of side length x from the corners of a 20- by 25-inch sheet of tin and bending up the sides (see figure below).

a) Write an equation for the volume V as a function of x. What is the domain of this function?



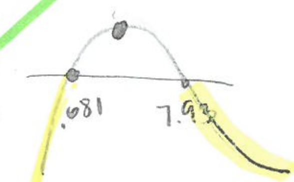
V = L·W·H

$V = (25-2x)(20-2x)(x)$

b) What size corner squares should be cut to yield a box with a volume of 300 in³?

$(25-2x)(20-2x)(x) = 300$ → graph

x = .681 inches & 7.93 inches



c) What size corner squares should be cut to yield a box with a volume more than 300 in³?

(.681, 7.93)

d) What size corner squares should be cut to yield a box with a volume of at most 300 in³?

(0, .681] or [7.93, 10)

e) How large should the square be to make the box hold as much as possible? What is the resulting volume?

x = 3.68 inches

V = 820.528 in³

1-5, 7, 9

Precalculus
Worksheet 2.8

$\boxed{m = \frac{-42}{40} = \frac{-21}{20}}$

1. $6(2m+3)\left(\frac{5}{6} - \frac{2m}{2m+3} = \frac{19}{6}\right)$

$5(2m+3) - 2m(6) = 19(2m+3)$

$10m + 15 - 12m = 38m + 57$

$-2m + 15 = 38m + 57$

$-42 = 40m$

2. $c\left(c - \frac{4}{c} = 3\right)$

$c^2 - 4 = 3c$

$c^2 - 3c - 4 = 0$

$(c-4)(c+1) = 0$

$\boxed{\begin{matrix} c = 4 \\ c = -1 \end{matrix}}$

3. $(x+2)(x-3)\left(\frac{3}{x+2} = \frac{4}{x-3}\right)$

$3(x-3) = 4(x+2)$

$3x - 9 = 4x + 8$

$-8 \quad -3x$

$\boxed{-17 = x}$

4. $(x+3)(x-3)\left(\frac{2}{x-3} - \frac{4}{x+3} = \frac{8}{(x+3)(x-3)}\right)$

$2(x+3) - 4(x-3) = 8$

$2x + 6 - 4x + 12 = 8$

$-2x + 18 = 8$

$-2x = -10$

$\boxed{x = 5}$

5. $\left(\frac{2x}{x-1} + \frac{5}{x+5} = 4\right)(x-1)(x+5)$

$\frac{2x(\cancel{x-1})(x+5)}{\cancel{x-1}} + \frac{5(x-1)(\cancel{x+5})}{(\cancel{x+5})} = 4(x-1)(x+5)$

$2x(x+5) + 5(x-1) = 4(x-1)(x+5)$

$2x^2 + 10x + 5x - 5 = 4(x^2 + 5x - x - 5)$

$2x^2 + 15x - 5 = 4x^2 + 16x - 20$

$0 = 2x^2 + x - 15 \to$ graph $\to$ quad form

$0 = (2x-5)(x+3) \to$ factor

$\boxed{x = 5/2 \quad x = -3}$

~~6. $\frac{2}{x+3} - \frac{3}{4-x} = \frac{2x-2}{x^2-x-12}$~~

7. $\left(\frac{2x}{x-3} - \frac{6}{x} = \frac{18}{x^2-3x}\right) x(x-3)$ (x^2-3x = $x(x-3)$)

$\frac{2x \cdot x \cdot (\cancel{x-3})}{\cancel{x-3}} - \frac{6 \cdot \cancel{x} \cdot (x-3)}{\cancel{x}} = \frac{18\cancel{x}(\cancel{x-3})}{\cancel{x}(\cancel{x-3})}$

$2x^2 - 6(x-3) = 18$

$2x^2 - 6x + 18 = 18$

$2x^2 - 6x = 0$

$2x(x-3) = 0$

$\boxed{\cancel{x = 0}}\ \boxed{\cancel{x = 3}}$ extraneous

no solution

~~8. $\frac{x-3}{x} + \frac{3}{x^2+x} = \frac{3}{x+1}$~~

9. $\left(\dfrac{3x}{x+5}+\dfrac{1}{x-2}=\dfrac{7}{\cancel{x^2+3x-10}}\right)(x+5)(x-2)$

$(x+5)(x-2)$

~~10. $\dfrac{10}{2y+8}+\dfrac{7y+8}{y^2-16}=\dfrac{-8}{2y-8}$~~

$$\frac{3x\cancel{(x+5)}(x-2)}{\cancel{x+5}}+\frac{1(x+5)\cancel{(x-2)}}{\cancel{x-2}}=\frac{7\cancel{(x+5)}\cancel{(x-2)}}{\cancel{(x+5)}\cancel{(x-2)}}$$

$3x(x-2)+1(x+5)=7$

$3x^2-6x+x+5=7$

$\rightarrow 3x^2-5x-2=0$

$(3x+1)(x-2)=0$

$\boxed{x=-1/3 \;|\; \cancel{x=2}}$

✓ extraneous

~~11. $\dfrac{1}{x(x-4)}+\dfrac{5-2x}{x^2-3x-4}=\dfrac{-5}{x(x+1)}$~~

~~12. Solve for x: $y=1+\dfrac{1}{1+x}$~~

~~13. The total electrical resistance R of two resistors connected in parallel with resistances R_1 and R_2 is given by~~

$$\cancel{\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}}$$

~~One of the resistors has a resistance of 2.3 ohms. Let x be the resistance of the second resistor.~~

~~a) Express the total resistance R as a function of x.~~

~~b) Find the resistance of the second resistor if the total resistance of the pair is 1.7 ohms.~~

14. Josh rode his bike 17 miles from his home in Columbus, and then traveled 53 miles by car from Columbus to Dayton. Assume the average rate of the car was 43 mph faster than the average rate of the bike.

a) Express the total time required to complete the 70-mi trip as a function of the rate x of the bike.

b) Find graphically the rate of the bike if the total time of the trip is 1 hr 40 min. Confirm algebraically.

15. Consider all the rectangles with an area of 200 m^2. Let x be the length of one side of such a rectangle.

a) Express the perimeter P as a function of x.

b) Find the dimensions of a rectangle whose perimeter is 70 m.

c) Find the dimensions of the rectangle that has the least perimeter. What is the least perimeter?